

# CHAPTER 1

## Introduction

### Notes for the Instructor

This chapter introduces the markets for futures, forward, and options contracts and explains the activities of hedgers, speculators, and arbitrageurs. Issues concerning futures contracts such as margin requirements, settlement procedures, the role of the clearinghouse, etc are covered in Chapter 2.

Some instructors prefer to avoid any mention of options until the material on linear products in Chapters 1 to 7 has been covered. I like to introduce students to options in the first class, even though they are not mentioned again for several classes. This is because most students find options to be the most interesting of the derivatives covered and I like students to be enthusiastic about the course early on.

The way in which the material in Chapter 1 is covered is likely to depend on the backgrounds of the students. If a course in investments is a prerequisite, Chapter 1 can be regarded as a review of material already familiar to the students and can be covered fairly quickly. If an investments course is not a prerequisite, more time may be required. Increasingly some aspects of derivatives markets are being covered in introductory corporate finance courses, accounting courses, strategy courses, etc. In many instances students are, therefore, likely to have had some exposure to the material in Chapter 1. I do not require an investments elective as a prerequisite for my elective on futures and options markets and find that  $1\frac{1}{2}$  to 2 hours is necessary for me to introduce the course and cover the material in Chapter 1.

To motivate students at the outset of the course, I discuss the growing importance of derivatives, how much well experts in the field are paid, etc. It is not uncommon for students who join derivatives groups, and are successful, to earn (including bonus) several hundred thousand dollars a year—or even \$1 million per year—three or four years after graduating.

Towards the end of the first class I usually produce a current newspaper and describe several traded futures and options. I then ask students to guess the quoted price. Sometimes votes are taken. This is an enjoyable exercise and forces students to think actively about the nature of the contracts and the determinants of price. It usually leads to a preliminary discussion of such issues as the relationship between a futures price and the corresponding spot price, the desirability of options being exercised early, why most options sell for more than their intrinsic value, etc.

While covering the Chapter 1 material, I treat futures as the same as forwards for the purposes of discussion. I try to avoid being drawn into a discussion of such issues as the mechanics of futures, margin requirements, daily settlement procedures, and so on until I am ready. These topics are covered in Chapter 2.

As will be evident from the slides that go with this chapter, I usually introduce students to a little of the Chapter 5 material during the first class. I discuss how arbitrage

arguments tie the futures price of gold to its spot price and why the futures price of a consumption commodity such as oil is not tied to its spot price in the same way. Problem 1.26 can be used to initiate the discussion.

I find that Problems 1.27 and 1.31 work well as an assignment questions. (1.31 has the advantage that it introduces students to DerivaGem early in the course.) Problem 1.28 usually generates a lively discussion. I sometimes ask students to consider it between the first and second class. We then discuss it at the beginning of the second class. Problems 1.29, 1.30, and 1.32 can be used either as assignment question or for class discussion.

## QUESTIONS AND PROBLEMS

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### Problem 1.1.

*What is the difference between a long forward position and a short forward position?*

When a trader enters into a long forward contract, she is agreeing to *buy* the underlying asset for a certain price at a certain time in the future. When a trader enters into a short forward contract, she is agreeing to *sell* the underlying asset for a certain price at a certain time in the future.

### Problem 1.2.

*Explain carefully the difference between hedging, speculation, and arbitrage.*

A trader is *hedging* when she has an exposure to the price of an asset and takes a position in a derivative to offset the exposure. In a *speculation* the trader has no exposure to offset. She is betting on the future movements in the price of the asset. *Arbitrage* involves taking a position in two or more different markets to lock in a profit.

### Problem 1.3.

*What is the difference between entering into a long forward contract when the forward price is \$50 and taking a long position in a call option with a strike price of \$50?*

In the first case the trader is obligated to buy the asset for \$50. (The trader does not have a choice.) In the second case the trader has an option to buy the asset for \$50. (The trader does not have to exercise the option.)

### Problem 1.4.

*Explain carefully the difference between selling a call option and buying a put option.*

Selling a call option involves giving someone else the right to buy an asset from you. It gives you a payoff of

$$-\max(S_T - K, 0) = \min(K - S_T, 0)$$

Buying a put option involves buying an option from someone else. It gives a payoff of

$$\max(K - S_T, 0)$$

In both cases the potential payoff is  $K - S_T$ . When you write a call option, the payoff is negative or zero. (This is because the counterparty chooses whether to exercise.) When you buy a put option, the payoff is zero or positive. (This is because you choose whether to exercise.)

**Problem 1.5.**

An investor enters into a short forward contract to sell 100,000 British pounds for U.S. dollars at an exchange rate of 1.9000 U.S. dollars per pound. How much does the investor gain or lose if the exchange rate at the end of the contract is (a) 1.8900 and (b) 1.9200?

- (a) The investor is obligated to sell pounds for 1.9000 when they are worth 1.8900. The gain is  $(1.9000 - 1.8900) \times 100,000 = \$1,000$ .
- (b) The investor is obligated to sell pounds for 1.9000 when they are worth 1.9200. The loss is  $(1.9200 - 1.9000) \times 100,000 = \$2,000$ .

**Problem 1.6.**

A trader enters into a short cotton futures contract when the futures price is 50 cents per pound. The contract is for the delivery of 50,000 pounds. How much does the trader gain or lose if the cotton price at the end of the contract is (a) 48.20 cents per pound; (b) 51.30 cents per pound?

- (a) The trader sells for 50 cents per pound something that is worth 48.20 cents per pound. Gain =  $(\$0.5000 - \$0.4820) \times 50,000 = \$900$ .
- (b) The trader sells for 50 cents per pound something that is worth 51.30 cents per pound. Loss =  $(\$0.5130 - \$0.5000) \times 50,000 = \$650$ .

**Problem 1.7.**

Suppose that you write a put contract with a strike price of \$40 and an expiration date in three months. The current stock price is \$41 and the contract is on 100 shares. What have you committed yourself to? How much could you gain or lose?

You have sold a put option. You have agreed to buy 100 shares for \$40 per share if the party on the other side of the contract chooses to exercise the right to sell for this price. The option will be exercised only when the price of stock is below \$40. Suppose, for example, that the option is exercised when the price is \$30. You have to buy at \$40 shares that are worth \$30; you lose \$10 per share, or \$1,000 in total. If the option is exercised when the price is \$20, you lose \$20 per share, or \$2,000 in total. The worst that can happen is that the price of the stock declines to almost zero during the three-month period. This highly unlikely event would cost you \$4,000. In return for the possible future losses, you receive the price of the option from the purchaser.

**Problem 1.8.**

What is the difference between the over-the-counter market and the exchange-traded market? What are the bid and offer quotes of a market maker in the over-the-counter market?

The over-the-counter market is a telephone- and computer-linked network of financial institutions, fund managers, and corporate treasurers where two participants can enter into any mutually acceptable contract. An exchange-traded market is a market organized by an exchange where traders either meet physically or communicate electronically and the contracts that can be traded have been defined by the exchange. When a market maker quotes a bid and an offer, the bid is the price at which the market maker is prepared to buy and the offer is the price at which the market maker is prepared to sell.

**Problem 1.9.**

*You would like to speculate on a rise in the price of a certain stock. The current stock price is \$29, and a three-month call with a strike of \$30 costs \$2.90. You have \$5,800 to invest. Identify two alternative strategies, one involving an investment in the stock and the other involving investment in the option. What are the potential gains and losses from each?*

One strategy would be to buy 200 shares. Another would be to buy 2,000 options. If the share price does well the second strategy will give rise to greater gains. For example, if the share price goes up to \$40 you gain  $[2,000 \times (\$40 - \$30)] - \$5,800 = \$14,200$  from the second strategy and only  $200 \times (\$40 - \$29) = \$2,200$  from the first strategy. However, if the share price does badly, the second strategy gives greater losses. For example, if the share price goes down to \$25, the first strategy leads to a loss of  $200 \times (\$29 - \$25) = \$800$ , whereas the second strategy leads to a loss of the whole \$5,800 investment. This example shows that options contain built in leverage.

**Problem 1.10.**

*Suppose that you own 5,000 shares worth \$25 each. How can put options be used to provide you with insurance against a decline in the value of your holding over the next four months?*

You could buy 5,000 put options (or 50 contracts) with a strike price of \$25 and an expiration date in 4 months. This provides a type of insurance. If at the end of 4 months the stock price proves to be less than \$25 you can exercise the options and sell the shares for \$25 each. The cost of this strategy is the price you pay for the put options.

**Problem 1.11.**

*When first issued, a stock provides funds for a company. Is the same true of a stock option? Discuss.*

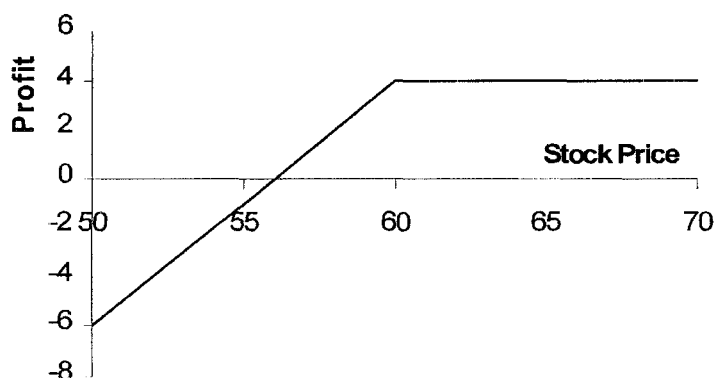
A stock option provides no funds for the company. It is a security sold by one trader to another. The company is not involved. By contrast, a stock when it is first issued is a claim sold by the company to investors and does provide funds for the company.

**Problem 1.12.**

*Explain why a forward contract can be used for either speculation or hedging.*

If a trader has an exposure to the price of an asset, she can hedge with a forward contract. If the exposure is such that the trader will gain when the price decreases and

option is in these circumstances less than the price received for the option. The option will be exercised if the stock price at maturity is less than \$60.00. Note that if the stock price is between \$56.00 and \$60.00 the seller of the option makes a profit even though the option is exercised. The profit from the short position is as shown in Figure S1.2.



**Figure S1.2** Profit from short position In Problem 1.14

**Problem 1.15.**

*It is May and a trader writes a September call option with a strike price of \$20. The stock price is \$18, and the option price is \$2. Describe the trader's cash flows if the option is held until September and the stock price is \$25 at that time.*

The trader receives an inflow of \$2 in May. Since the option is exercised, the trader also has an outflow of \$5 in September. The \$2 is the cash received from the sale of the option. The \$5 is the result of buying the stock for \$25 in September and selling it to the purchaser of the option for \$20. One contract consists of 100 options and so the cash flows for a contract are multiplied by 100.

**Problem 1.16.**

*A trader writes a December put option with a strike price of \$30. The price of the option is \$4. Under what circumstances does the trader make a gain?*

The trader makes a gain if the price of the stock is above \$26 in December. (This ignores the time value of money.)

**Problem 1.17.**

*A company knows that it is due to receive a certain amount of a foreign currency in four months. What type of option contract is appropriate for hedging?*

A long position in a four-month put option can provide insurance against the exchange rate falling below the strike price. It ensures that the foreign currency can be sold for at least the strike price.

**Problem 1.18.**

A United States company expects to have to pay 1 million Canadian dollars in six months. Explain how the exchange rate risk can be hedged using (a) a forward contract; (b) an option.

The company could enter into a long forward contract to buy 1 million Canadian dollars in six months. This would have the effect of locking in an exchange rate equal to the current forward exchange rate. Alternatively the company could buy a call option giving it the right (but not the obligation) to purchase 1 million Canadian dollar at a certain exchange rate in six months. This would provide insurance against a strong Canadian dollar in six months while still allowing the company to benefit from a weak Canadian dollar at that time.

**Problem 1.19.**

A trader enters into a short forward contract on 100 million yen. The forward exchange rate is \$0.0080 per yen. How much does the trader gain or lose if the exchange rate at the end of the contract is (a) \$0.0074 per yen; (b) \$0.0091 per yen?

- (a) The trader sells 100 million yen for \$0.0080 per yen when the exchange rate is \$0.0074 per yen. The gain is  $100 \times 0.0006$  millions of dollars or \$60,000.
- (b) The trader sells 100 million yen for \$0.0080 per yen when the exchange rate is \$0.0091 per yen. The loss is  $100 \times 0.0011$  millions of dollars or \$110,000.

**Problem 1.20.**

The Chicago Board of Trade offers a futures contract on long-term Treasury bonds. Characterize the traders likely to use this contract.

Most traders who use the contract will wish to do one of the following:

- (a) Hedge their exposure to long-term interest rates
- (b) Speculate on the future direction of long-term interest rates
- (c) Arbitrage between cash and futures markets

This contract is discussed in Chapter 6.

**Problem 1.21.**

“Options and futures are zero-sum games.” What do you think is meant by this statement?

The statement means that the gain (loss) to the party with a short position in an option is always equal to the loss (gain) to the party with the long position. The sum of the gains is zero.

**Problem 1.22.**

Describe the profit from the following portfolio: a long forward contract on an asset and a long European put option on the asset with the same maturity as the forward contract and a strike price that is equal to the forward price of the asset at the time the portfolio is set up.

The terminal value of the long forward contract is:

$$S_T - F_0$$

where  $S_T$  is the price of the asset at maturity and  $F_0$  is the forward price of the asset at the time the portfolio is set up. (The delivery price in the forward contract is  $F_0$ .)

The terminal value of the put option is:

$$\max(F_0 - S_T, 0)$$

The terminal value of the portfolio is therefore

$$\begin{aligned} S_T - F_0 + \max(F_0 - S_T, 0) \\ = \max(0, S_T - F_0) \end{aligned}$$

This is the same as the terminal value of a European call option with the same maturity as the forward contract and an exercise price equal to  $F_0$ . This result is illustrated in the Figure S1.3. The profit equals the terminal value less the amount paid for the option.

**Problem 1.23.**

In the 1980s, Bankers Trust developed *index currency option notes (ICONs)*. These are bonds in which the amount received by the holder at maturity varies with a foreign exchange rate. One example was its trade with the Long Term Credit Bank of Japan. The ICON specified that if the yen–U.S. dollar exchange rate,  $S_T$ , is greater than 169 yen per dollar at maturity (in 1995), the holder of the bond receives \$1,000. If it is less than 169 yen per dollar, the amount received by the holder of the bond is

$$1,000 - \max \left[ 0, 1,000 \left( \frac{169}{S_T} - 1 \right) \right]$$

When the exchange rate is below 84.5, nothing is received by the holder at maturity. Show that this ICON is a combination of a regular bond and two options.

Suppose that the yen exchange rate (yen per dollar) at maturity of the ICON is  $S_T$ . The payoff from the ICON is

$$\begin{aligned} & 1,000 && \text{if} && S_T > 169 \\ & 1,000 - 1,000 \left( \frac{169}{S_T} - 1 \right) && \text{if} && 84.5 \leq S_T \leq 169 \\ & 0 && \text{if} && S_T < 84.5 \end{aligned}$$

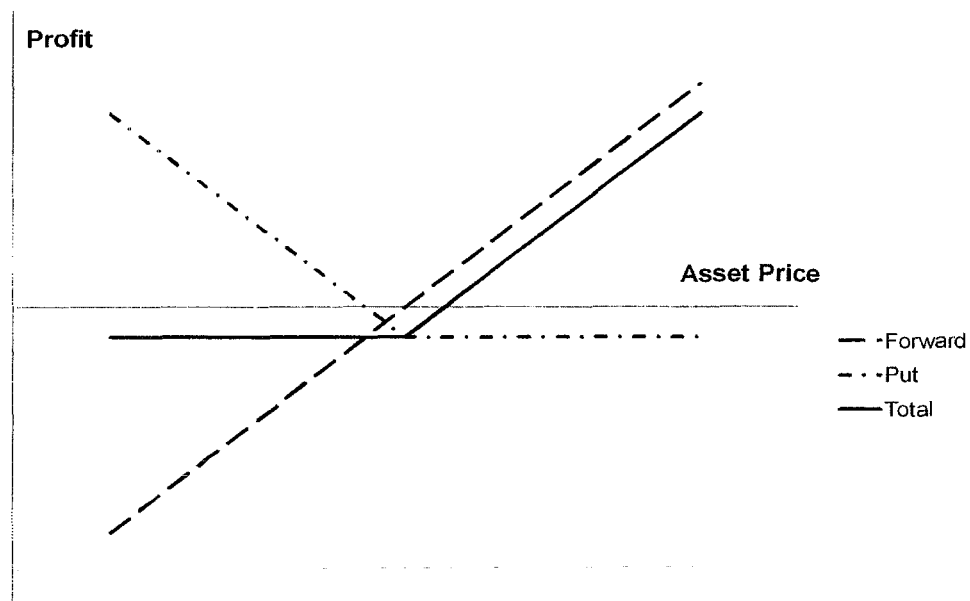


Figure S1.3 Profit from portfolio in Problem 1.22

When  $84.5 \leq S_T \leq 169$  the payoff can be written

$$2,000 - \frac{169,000}{S_T}$$

The payoff from an ICON is the payoff from:

- (a) A regular bond
- (b) A short position in call options to buy 169,000 yen with an exercise price of 1/169
- (c) A long position in call options to buy 169,000 yen with an exercise price of 1/84.5

This is demonstrated by the following table

	Terminal Value of Regular Bond	Terminal Value of Short Calls	Terminal Value of Long Calls	Terminal Value of Whole Position
$S_T > 169$	1,000	0	0	1000
$84.5 \leq S_T \leq 169$	1000	$-169,000 \left( \frac{1}{S_T} - \frac{1}{169} \right)$	0	$2000 - \frac{169,000}{S_T}$
$S_T < 84.5$	1000	$-169,000 \left( \frac{1}{S_T} - \frac{1}{169} \right)$	$169,000 \left( \frac{1}{S_T} - \frac{1}{84.5} \right)$	0



**Problem 1.24.**

On July 1, 2008, a company enters into a forward contract to buy 10 million Japanese yen on January 1, 2009. On September 1, 2008, it enters into a forward contract to sell 10 million Japanese yen on January 1, 2009. Describe the payoff from this strategy.

Suppose that the forward price for the contract entered into on July 1, 2008 is  $F_1$  and that the forward price for the contract entered into on September 1, 2008 is  $F_2$  with both  $F_1$  and  $F_2$  being measured as dollars per yen. If the value of one Japanese yen (measured in U.S. dollars) is  $S_T$  on January 1, 2009, then the value of the first contract (in millions of dollars) at that time is

$$10(S_T - F_1)$$

while the value of the second contract (per yen sold) at that time is:

$$10(F_2 - S_T)$$

The total payoff from the two contracts is therefore

$$10(S_T - F_1) + 10(F_2 - S_T) = 10(F_2 - F_1)$$

Thus if the forward price for delivery on January 1, 2009 increases between July 1, 2008 and September 1, 2008 the company will make a profit.

**Problem 1.25.**

Suppose that USD-sterling spot and forward exchange rates are as follows:

Spot	2.0080
90-day forward	2.0056
180-day forward	2.0018

What opportunities are open to an arbitrageur in the following situations?

- A 180-day European call option to buy £1 for \$1.97 costs 2 cents.
- A 90-day European put option to sell £1 for \$2.04 costs 2 cents.

(a) The trader buys a 180-day call option and takes a short position in a 180-day forward contract. If  $S_T$  is the terminal spot rate, the profit from the call option is

$$\max(S_T - 1.97, 0) - 0.02$$

The profit from the short forward contract is

$$2.0018 - S_T$$

The profit from the strategy is therefore

$$\max(S_T - 1.97, 0) - 0.02 + 2.0018 - S_T$$

or

$$\max(S_T - 1.97, 0) + 1.9818 - S_T$$

This is

$$\begin{array}{ll} 1.9818 - S_T & \text{when } S_T < 1.97 \\ 0.0118 & \text{when } S_T > 1.97 \end{array}$$

This shows that the profit is always positive. The time value of money has been ignored in these calculations. However, when it is taken into account the strategy is still likely to be profitable in all circumstances. (We would require an extremely high interest rate for \$0.0118 interest to be required on an outlay of \$0.02 over a 180-day period.)

(b) The trader buys 90-day put options and takes a long position in a 90 day forward contract. If  $S_T$  is the terminal spot rate, the profit from the put option is

$$\max(2.04 - S_T, 0) - 0.020$$

The profit from the long forward contract is

$$S_T - 2.0056$$

The profit from this strategy is therefore

$$\max(2.04 - S_T, 0) - 0.020 + S_T - 2.0056$$

or

$$\max(2.04 - S_T, 0) + S_T - 2.0256$$

This is

$$\begin{array}{ll} S_T - 2.0256 & \text{when } S_T > 2.04 \\ 0.0144 & \text{when } S_T < 2.04 \end{array}$$

The profit is therefore always positive. Again, the time value of money has been ignored but is unlikely to affect the overall profitability of the strategy. (We would require interest rates to be extremely high for \$0.0144 interest to be required on an outlay of \$0.02 over a 90-day period.)

## ASSIGNMENT QUESTIONS

### Problem 1.26.

*The price of gold is currently \$600 per ounce. The forward price for delivery in one year is \$800. An arbitrageur can borrow money at 10% per annum. What should the arbitrageur do? Assume that the cost of storing gold is zero and that gold provides no income.*

The arbitrageur could borrow money to buy 100 ounces of gold today and short futures contracts on 100 ounces of gold for delivery in one year. This means that gold is purchased for \$600 per ounce and sold for \$800 per ounce. The return (33.3% per annum) is far greater than the 10% cost of the borrowed funds. This is such a profitable opportunity that the arbitrageur should buy as many ounces of gold as possible and short futures contracts on

the same number of ounces. Unfortunately arbitrage opportunities as profitable as this rarely arise in practice.

**Problem 1.27.**

*The current price of a stock is \$94, and three-month European call options with a strike price of \$95 currently sell for \$4.70. An investor who feels that the price of the stock will increase is trying to decide between buying 100 shares and buying 2,000 call options (= 20 contracts). Both strategies involve an investment of \$9,400. What advice would you give? How high does the stock price have to rise for the option strategy to be more profitable?*

The investment in call options entails higher risks but can lead to higher returns. If the stock price stays at \$94, an investor who buys call options loses \$9,400 whereas an investor who buys shares neither gains nor loses anything. If the stock price rises to \$120, the investor who buys call options gains

$$2000 \times (120 - 95) - 9400 = \$40,600$$

An investor who buys shares gains

$$100 \times (120 - 94) = \$2,600$$

The strategies are equally profitable if the stock price rises to a level,  $S$ , where

$$100 \times (S - 94) = 2000(S - 95) - 9400$$

or

$$S = 100$$

The option strategy is therefore more profitable if the stock price rises above \$100.

**Problem 1.28.**

*On September 12, 2006, an investor owns 100 Intel shares. As indicated in Table 1.2 the share price is \$19.56 and a January put option with a strike price \$17.50 costs \$0.475. The investor is comparing two alternatives to limit downside risk. The first is to buy one January put option contract with a strike price of \$17.50. The second involves instructing a broker to sell the 100 shares as soon as Intel's price reaches 2\$17.50. Discuss the advantages and disadvantages of the two strategies.*

The second alternative involves what is known as a stop or stop-loss order. It costs nothing and ensures that \$1,750, or close to \$1,750 is realized for the holding in the event the stock price ever falls to \$17.50. The put option costs \$47.50 and guarantees that the holding can be sold for \$1,750 any time up to January. If the stock price falls marginally below \$17.50 and then rises the option will not be exercised, but the stop-loss order will lead to the holding being liquidated. There are some circumstances where the put option alternative leads to a better outcome and some circumstances where the stop-loss order leads to a better outcome. If the stock price ends up below \$17.50, the stop-loss order

alternative leads to a better outcome because the cost of the option is avoided. If the stock price falls to \$17 in October and then rises to \$30 by January, the put option alternative leads to a better outcome. The investor is paying \$47.50 for the chance to benefit from this second type of outcome.

**Problem 1.29.**

A bond issued by Standard Oil some time ago worked as follows. The holder received no interest. At the bond's maturity the company promised to pay \$1,000 plus an additional amount based on the price of oil at that time. The additional amount was equal to the product of 170 and the excess (if any) of the price of a barrel of oil at maturity over \$25. The maximum additional amount paid was \$2,550 (which corresponds to a price of \$40 per barrel). Show that the bond is a combination of a regular bond, a long position in call options on oil with a strike price of \$25, and a short position in call options on oil with a strike price of \$40.

Suppose  $S_T$  is the price of oil at the bond's maturity. In addition to \$1000 the Standard Oil bond pays:

$$\begin{aligned} S_T < \$25 & : & 0 \\ \$25 < S_T < \$40 & : & 170(S_T - 25) \\ S_T > \$40 & : & 2,550 \end{aligned}$$

This is the payoff from 170 call options on oil with a strike price of 25 less the payoff from 170 call options on oil with a strike price of 40. The bond is therefore equivalent to a regular bond plus a long position in 170 call options on oil with a strike price of \$25 plus a short position in 170 call options on oil with a strike price of \$40. The investor has what is termed a bull spread on oil. This is discussed in Chapter 10.

**Problem 1.30.**

Suppose that in the situation of Table 1.1 a corporate treasurer said: "I will have £1 million to sell in six months. If the exchange rate is less than 2.02 I want you to give me 2.02. If it is greater than 2.09 I will accept 2.09. If the exchange rate is between 2.02 and 2.09 I will sell the sterling for the exchange rate." How could you use options to satisfy the treasurer?

You sell the Treasurer a put option on GBP with a strike price of 2.02 and buy from the treasurer a call option on GBP with a strike price of 2.09. Both options are on one million pounds and have a maturity of six months. This is known as a range forward contract.

**Problem 1.31.**

Describe how foreign currency options can be used for hedging in the situation considered in Section 1.7 so that (a) ImportCo is guaranteed that its exchange rate will be less than 2.0700, and (b) ExportCo is guaranteed that its exchange rate will be at least 2.0400. Use DerivaGem to calculate the cost of setting up the hedge in each case assuming that the exchange rate volatility is 12%, interest rates in the United States are 5% and

interest rates in Britain are 5.7%. Assume that the current exchange rate is the average of the bid and offer in Table 1.1.

ImportCo should buy three-month call options on £10 million with a strike price of 2.0700. ExportCo should buy three-month put options on £10 million with a strike price of 2.0400. In this case the foreign exchange rate is 2.0560 (the average of the bid and offer quotes in Table 1.1.), the (domestic) risk-free rate is 5%, the foreign risk-free rate is 5.7%, the volatility is 12%, and the time to exercise is 0.25. Using the Equity\_FX\_Index\_Futures\_Options worksheet in the DerivaGem Options Calculator select Currency as the underlying and Analytic European as the option type. The software shows that a call with a strike price of 2.07 is worth 0.0405 and a put with a strike of 2.04 is worth 0.0425. This means that the hedging would cost  $0.0405 \times 10,000,000$  or \$405,000 for ImportCo and  $0.0425 \times 10,000,000$  or \$425,000 for ExportCo.

**Problem 1.32.**

*A trader buys a European call option and sells a European put option. The options have the same underlying asset, strike price and maturity. Describe the trader's position. Under what circumstances does the price of the call equal the price of the put?*

The trader has a long European call option with strike price  $K$  and a short European put option with strike price  $K$ . Suppose the price of the underlying asset at the maturity of the option is  $S_T$ . If  $S_T > K$ , the call option is exercised by the investor and the put option expires worthless. The payoff from the portfolio is  $S_T - K$ . If  $S_T < K$ , the call option expires worthless and the put option is exercised against the investor. The cost to the investor is  $K - S_T$ . Alternatively we can say that the payoff to the investor is  $S_T - K$  (a negative amount). In all cases, the payoff is  $S_T - K$ , the same as the payoff from the forward contract. The trader's position is equivalent to a forward contract with delivery price  $K$ .

Suppose that  $F$  is the forward price. If  $K = F$ , the forward contract that is created has zero value. Because the forward contract is equivalent to a long call and a short put, this shows that the price of a call equals the price of a put when the strike price is  $F$ .

# CHAPTER 2

## Mechanics of Futures Markets

### Notes for the Instructor

This chapter explains the functioning of futures markets. I do not spend a great deal of time in class going over most of the details of how futures markets work. I let students read these for themselves. But I do find it worth spending some time going through Table 2.1 to explain the way in which margin accounts work. I also draw students' attention to the patterns of futures prices in Figure 2.2. After the essentials of the operations of futures markets have been explained, I ask students to consider Problem 2.22 in class because I find that this often reveals gaps in their understanding. I usually use about  $1\frac{1}{2}$  hours to cover the material in the chapter.

There are many ways of making a discussion of futures markets fun. An easy-to-organize trading game that was explained to me by a Wall Street training manager works as follows. The instructor chooses two students to keep trading records on the front board and divides the rest of the students into about ten groups. Each group is given an identifier (e.g., A, B, C, etc) and a card with the identifier shown in big letters. They display the card when they want to make trades. The instructor chooses a seven-digit telephone number, but does not reveal this to students. The groups trade the sum of digits of the telephone number by entering long or short positions. For example, group B might bid (i.e. offer to buy) at 35. If this is accepted by another group (say group D), the record keepers show that B is long one contract at 35 and D is short one contract at 35. (If the actual sum of digits is 32, B is  $-3$  on the trade and D is  $+3$ . The instructor controls the trading, asks for bids or offers as appropriate, and shouts trades to the record keepers. Every two minutes the instructor reveals one of the digits of the number. This game nearly always works very well for me. Trading typically starts slowly and then becomes very intense. The game gives students a sense of what futures trading is like. I insist that they use the words bid and offer rather than buy and sell.) It shows how prices are formed in markets. (After the game is over we discuss how the market price moved during the game.) The records also usually show different trading strategies. Some groups are usually speculators (all trades are long or all are short) and others are like day traders (e.g., buy at 35, sell at 36, buy at 38, sell at 39, etc). I point out to students that we need both types of traders to make the market work.

There are many stories that can be told about futures markets. Students are often interested in attempts to corner markets. I explain that the Hunt brothers' exploits in the silver market (See footnote 2 in 2.8) bankrupted them because the exchange forced them to close out their positions prior to the delivery month and as a result the price dropped. The brothers tried unsuccessfully to sue the exchange.

Business Snapshot 2.1 is an amusing story that I have often told in class. Business Snapshot 2.2 (on Long Term Capital Management) fits in well when the operation of margin accounts is being explained.

I sometimes use Problems 2.26, 2.27, and 2.28 as short hand-in assignments. Problem 2.29 is more challenging, but can be a good learning experience for students.

## QUESTIONS AND PROBLEMS

### Problem 2.1.

*Distinguish between the terms open interest and trading volume.*

The *open interest* of a futures contract at a particular time is the total number of long positions outstanding. (Equivalently, it is the total number of short positions outstanding.) The *trading volume* during a certain period of time is the number of contracts traded during this period.

### Problem 2.2.

*What is the difference between a local and a commission broker?*

A *commission broker* trades on behalf of a client and charges a commission. A *local* trades on his or her own behalf.

### Problem 2.3.

*Suppose that you enter into a short futures contract to sell July silver for \$10.20 per ounce on the New York Commodity Exchange. The size of the contract is 5,000 ounces. The initial margin is \$4,000, and the maintenance margin is \$3,000. What change in the futures price will lead to a margin call? What happens if you do not meet the margin call?*

There will be a margin call when \$1,000 has been lost from the margin account. This will occur when the price of silver increases by  $1,000/5,000 = \$0.20$ . The price of silver must therefore rise to \$10.40 per ounce for there to be a margin call. If the margin call is not met, your broker closes out your position.

### Problem 2.4.

*Suppose that in September 2009 a company takes a long position in a contract on May 2010 crude oil futures. It closes out its position in March 2010. The futures price (per barrel) is \$68.30 when it enters into the contract, \$70.50 when it closes out its position, and \$69.10 at the end of December 2009. One contract is for the delivery of 1,000 barrels. What is the company's total profit? When is it realized? How is it taxed if it is (a) a hedger and (b) a speculator? Assume that the company has a December 31 year-end.*

The total profit is  $(\$70.50 - \$68.30) \times 1,000 = \$2,200$ . Of this  $(\$69.10 - \$68.30) \times 1,000 = \$800$  is realized on a day-by-day basis between September 2009 and December 31, 2009. A further  $(\$70.50 - \$69.10) \times 1,000 = \$1,400$  is realized on a day-by-day basis between January 1, 2009, and March 2010. A hedger would be taxed on the whole profit of \$2,200 in 2010. A speculator would be taxed on \$800 in 2009 and \$1,400 in 2010.

**Problem 2.5.**

*What does a stop order to sell at \$2 mean? When might it be used? What does a limit order to sell at \$2 mean? When might it be used?*

A *stop order* to sell at \$2 is an order to sell at the best available price once a price of \$2 or less is reached. It could be used to limit the losses from an existing long position. A *limit order* to sell at \$2 is an order to sell at a price of \$2 or more. It could be used to instruct a broker that a short position should be taken, providing it can be done at a price more favorable than \$2.

**Problem 2.6.**

*What is the difference between the operation of the margin accounts administered by a clearinghouse and those administered by a broker?*

The margin account administered by the clearinghouse is marked to market daily, and the clearinghouse member is required to bring the account back up to the prescribed level daily. The margin account administered by the broker is also marked to market daily. However, the account does not have to be brought up to the initial margin level on a daily basis. It has to be brought up to the initial margin level when the balance in the account falls below the maintenance margin level. The maintenance margin is usually about 75% of the initial margin.

**Problem 2.7.**

*What differences exist in the way prices are quoted in the foreign exchange futures market, the foreign exchange spot market, and the foreign exchange forward market?*

In futures markets, prices are quoted as the number of U.S. dollars per unit of foreign currency. Spot and forward rates are quoted in this way for the British pound, euro, Australian dollar, and New Zealand dollar. For other major currencies, spot and forward rates are quoted as the number of units of foreign currency per U.S. dollar.

**Problem 2.8.**

*The party with a short position in a futures contract sometimes has options as to the precise asset that will be delivered, where delivery will take place, when delivery will take place, and so on. Do these options increase or decrease the futures price? Explain your reasoning.*

These options make the contract less attractive to the party with the long position and more attractive to the party with the short position. They therefore tend to reduce the futures price.

**Problem 2.9.**

*What are the most important aspects of the design of a new futures contract?*

The most important aspects of the design of a new futures contract are the specification of the underlying asset, the size of the contract, the delivery arrangements, and the delivery months.



**Problem 2.10.**

*Explain how margins protect investors against the possibility of default.*

A margin is a sum of money deposited by an investor with his or her broker. It acts as a guarantee that the investor can cover any losses on the futures contract. The balance in the margin account is adjusted daily to reflect gains and losses on the futures contract. If losses are above a certain level, the investor is required to deposit a further margin. This system makes it unlikely that the investor will default. A similar system of margins makes it unlikely that the investor's broker will default on the contract it has with the clearinghouse member and unlikely that the clearinghouse member will default with the clearinghouse.

**Problem 2.11.**

*A trader buys two long July futures contracts on orange juice. Each contract is for the delivery of 15,000 pounds. The current futures price is 160 cents per pound, the initial margin is \$6,000 per contract, and the maintenance margin is \$4,500 per contract. What price change would lead to a margin call? Under what circumstances could \$2,000 be withdrawn from the margin account?*

There is a margin call if \$1,500 is lost on one contract. This happens if the futures price of orange juice falls by 10 cents to 150 cents per lb. \$2,000 can be withdrawn from the margin account if there is a gain on one contract of \$1,000. This will happen if the futures price rises by 6.67 cents to 166.67 cents per lb.

**Problem 2.12.**

*Show that if the futures price of a commodity is greater than the spot price during the delivery period there is an arbitrage opportunity. Does an arbitrage opportunity exist if the futures price is less than the spot price? Explain your answer.*

If the futures price is greater than the spot price during the delivery period, an arbitrageur buys the asset, shorts a futures contract, and makes delivery for an immediate profit. If the futures price is less than the spot price during the delivery period, there is no similar perfect arbitrage strategy. An arbitrageur can take a long futures position but cannot force immediate delivery of the asset. The decision on when delivery will be made is made by the party with the short position. Nevertheless companies interested in acquiring the asset will find it attractive to enter into a long futures contract and wait for delivery to be made.

**Problem 2.13.**

*Explain the difference between a market-if-touched order and a stop order.*

A market-if-touched order is executed at the best available price after a trade occurs at a specified price or at a price more favorable than the specified price. A stop order is executed at the best available price after there is a bid or offer at the specified price or at a price less favorable than the specified price.

**Problem 2.14.**

*Explain what a stop-limit order to sell at 20.30 with a limit of 20.10 means.*

A stop-limit order to sell at 20.30 with a limit of 20.10 means that as soon as there is a bid at 20.30 the contract should be sold providing this can be done at 20.10 or a higher price.

**Problem 2.15.**

*At the end of one day a clearinghouse member is long 100 contracts, and the settlement price is \$50,000 per contract. The original margin is \$2,000 per contract. On the following day the member becomes responsible for clearing an additional 20 long contracts, entered into at a price of \$51,000 per contract. The settlement price at the end of this day is \$50,200. How much does the member have to add to its margin account with the exchange clearinghouse?*

The clearinghouse member is required to provide  $20 \times \$2,000 = \$40,000$  as initial margin for the new contracts. There is a gain of  $(50,200 - 50,000) \times 100 = \$20,000$  on the existing contracts. There is also a loss of  $(51,000 - 50,200) \times 20 = \$16,000$  on the new contracts. The member must therefore add

$$40,000 - 20,000 + 16,000 = \$36,000$$

to the margin account.

**Problem 2.16.**

*On July 1, 2009, a Japanese company enters into a forward contract to buy \$1 million on January 1, 2010. On September 1, 2009, it enters into a forward contract to sell \$1 million on January 1, 2010. Describe the profit or loss the company will make in yen as a function of the forward exchange rates on July 1, 2009 and September 1, 2009.*

Suppose  $F_1$  and  $F_2$  are the forward exchange rates for the contracts entered into July 1, 2009 and September 1, 2009, and  $S$  is the spot rate on January 1, 2010. (All exchange rates are measured as yen per dollar). The payoff from the first contract is  $(S - F_1)$  million yen and the payoff from the second contract is  $(F_2 - S)$  million yen. The total payoff is therefore  $(S - F_1) + (F_2 - S) = (F_2 - F_1)$  million yen.

**Problem 2.17.**

*The forward price on the Swiss franc for delivery in 45 days is quoted as 1.2500. The futures price for a contract that will be delivered in 45 days is 0.7980. Explain these two quotes. Which is more favorable for an investor wanting to sell Swiss francs?*

The 1.2500 forward quote is the number of Swiss francs per dollar. The 0.7980 futures quote is the number of dollars per Swiss franc. When quoted in the same way as the futures price the forward price is  $1/1.2500 = 0.8000$ . The Swiss franc is therefore more valuable in the forward market than in the futures market. The forward market is therefore more attractive for an investor wanting to sell Swiss francs.

**Problem 2.18.**

Suppose you call your broker and issue instructions to sell one July hogs contract. Describe what happens.

Hog futures are traded on the Chicago Mercantile Exchange. (See Table 2.2). The broker will request some initial margin. The order will be relayed by telephone to your broker's trading desk on the floor of the exchange (or to the trading desk of another broker).

It will be sent by messenger to a commission broker who will execute the trade according to your instructions. Confirmation of the trade eventually reaches you. If there are adverse movements in the futures price your broker may contact you to request additional margin.

**Problem 2.19.**

*"Speculation in futures markets is pure gambling. It is not in the public interest to allow speculators to trade on a futures exchange."* Discuss this viewpoint.

Speculators are important market participants because they add liquidity to the market. However, contracts must be useful for hedging as well as speculation. This is because regulators generally only approve contracts when they are likely to be of interest to hedgers as well as speculators.

**Problem 2.20.**

Identify the contracts that have the highest open interest in Table 2.2.

The table does not show contracts for all maturities. In the Metals and Petroleum category it appears that crude oil has the highest open interest. In the agricultural category it appears that corn has the highest open interest.

**Problem 2.21.**

What do you think would happen if an exchange started trading a contract in which the quality of the underlying asset was incompletely specified?

The contract would not be a success. Parties with short positions would hold their contracts until delivery and then deliver the cheapest form of the asset. This might well be viewed by the party with the long position as garbage! Once news of the quality problem became widely known no one would be prepared to buy the contract. This shows that futures contracts are feasible only when there are rigorous standards within an industry for defining the quality of the asset. Many futures contracts have in practice failed because of the problem of defining quality.

**Problem 2.22.**

*"When a futures contract is traded on the floor of the exchange, it may be the case that the open interest increases by one, stays the same, or decreases by one."* Explain this statement.

If both sides of the transaction are entering into a new contract, the open interest increases by one. If both sides of the transaction are closing out existing positions, the

open interest decreases by one. If one party is entering into a new contract while the other party is closing out an existing position, the open interest stays the same.

**Problem 2.23.**

Suppose that on October 24, 2009, a company sells one April 2010 live-cattle futures contract. It closes out its position on January 21, 2010. The futures price (per pound) is 91.20 cents when it enters into the contract, 88.30 cents when it close out its position, and 88.80 cents at the end of December 2009. One contract is for the delivery of 40,000 pounds of cattle. What is the total profit? How is it taxed if the company is (a) a hedger and (b) a speculator? Assume that the company has a December 31 year end.

The total profit is

$$40,000 \times (0.9120 - 0.8830) = \$1,160$$

If you are a hedger this is all taxed in 2009. If you are a speculator

$$40,000 \times (0.9120 - 0.8880) = \$960$$

is taxed in 2009 and

$$40,000 \times (0.8880 - 0.8830) = \$200$$

is taxed in 2010.

**Problem 2.24.**

A cattle farmer expects to have 120,000 pounds of live cattle to sell in three months. The live-cattle futures contract on the Chicago Mercantile Exchange is for the delivery of 40,000 pounds of cattle. How can the farmer use the contract for hedging? From the farmer's viewpoint, what are the pros and cons of hedging?

The farmer can short 3 contracts that have 3 months to maturity. If the price of cattle falls, the gain on the futures contract will offset the loss on the sale of the cattle. If the price of cattle rises, the gain on the sale of the cattle will be offset by the loss on the futures contract. Using futures contracts to hedge has the advantage that it can at no cost reduce risk to almost zero. Its disadvantage is that the farmer no longer gains from favorable movements in cattle prices.

**Problem 2.25.**

It is now July 2008. A mining company has just discovered a small deposit of gold. It will take six months to construct the mine. The gold will then be extracted on a more or less continuous basis for one year. Futures contracts on gold are available on the New York Commodity Exchange. There are delivery months every two months from August 2008 to December 2009. Each contract is for the delivery of 100 ounces. Discuss how the mining company might use futures markets for hedging.

The mining company can estimate its production on a month by month basis. It can then short futures contracts to lock in the price received for the gold. For example, if a

total of 3,000 ounces are expected to be produced in January 2009 and February 2009, the price received for this production can be hedged by shorting a total of 30 February 2009 contracts.

## ASSIGNMENT QUESTIONS

### Problem 2.26.

*A company enters into a short futures contract to sell 5,000 bushels of wheat for 450 cents per bushel. The initial margin is \$3,000 and the maintenance margin is \$2,000. What price change would lead to a margin call? Under what circumstances could \$1,500 be withdrawn from the margin account?*

There is a margin call if \$1000 is lost on the contract. This will happen if the price of wheat futures rises by 20 cents from 450 cents to 470 cents per bushel. \$1500 can be withdrawn if the futures price falls by 30 cents to 420 cents per bushel.

### Problem 2.27.

*Suppose that there are no storage costs for crude oil and the interest rate for borrowing or lending is 5% per annum. How could you make money on January 8, 2007 by trading June 2007 and December 2007 contracts on crude oil? Use Table 2.2.*

The June 2007 settlement price for oil is \$60.01 per barrel. The December 2007 settlement price for oil is \$62.94 per barrel. You could go long one June 2007 oil contract and short one December 2007 contract. In June 2007 you take delivery of the oil borrowing \$60.01 per barrel at 5% to meet cash outflows. The interest accumulated in six months is about  $60.01 \times 0.05 \times 0.5$  or \$1.50. In December the oil is sold for \$62.94 and  $60.01 + 1.50 =$  \$61.51 per barrel has to be repaid on the loan. The strategy therefore leads to a profit of  $62.94 - 61.51$  or \$1.43 per barrel. Note that this profit is independent of the actual price of oil in June 2007 or December 2007. It will be slightly affected by the daily settlement procedures.

### Problem 2.28.

*What position is equivalent to a long forward contract to buy an asset at  $K$  on a certain date and a put option to sell it for  $K$  on that date.*

Suppose an investor has a long European call option with strike price  $K$  and a short European put option with strike price  $K$ . Suppose the price of the underlying asset at the maturity of the option is  $S_T$ . If  $S_T > K$ , the call option is exercised by the investor and the put option expires worthless. The payoff from the portfolio is  $S_T - K$ . If  $S_T < K$ , the call option expires worthless and the put option is exercised against the investor. The cost to the investor is  $K - S_T$ . Alternatively we can say that the payoff to the investor is  $S_T - K$  (a negative amount). In all cases, the payoff is  $S_T - K$ , the same as the payoff from the forward contract.

Suppose that  $F$  is the forward price. If  $K = F$ , the forward contract that is created has zero value. Because the forward contract is equivalent to a long call and a short put, this shows that the price of a call equals the price of a put when the strike price is  $F$ .

**Problem 2.29.**

The author's Web page ([www.rotman.utoronto.ca/~hull/data](http://www.rotman.utoronto.ca/~hull/data)) contains daily closing prices for crude oil futures and gold futures contracts. (Both contracts are traded on NYMEX.) You are required to download the data and answer the following:

- a. How high do the maintenance margin levels for oil and gold have to be set so that there is a 1% chance that an investor with a balance slightly above the maintenance margin level on a particular day has a negative balance two days later? How high do they have to be for a 0.1% chance? Assume daily price changes are normally distributed with mean zero. Explain why NYMEX might be interested in this calculation.
- b. Imagine an investor who starts with a long position in the oil contract at the beginning of the period covered by the data and keeps the contract for the whole of the period of time covered by the data. Margin balances in excess of the initial margin are withdrawn. Use the maintenance margin you calculated in part (a) for a 1% risk level and assume that the maintenance margin is 75% of the initial margin. Calculate the number of margin calls and the number of times the investor has a negative margin balance. Assume that all margin calls are met in your calculations. Repeat the calculations for an investor who starts with a short position in the gold contract.

- (a) For gold the standard deviation of daily changes is \$2.77 per ounce or \$277 per contract. For a 1% risk this means that the maintenance margin should be set at  $277 \times \sqrt{2} \times 2.33 = 912$ . For a 0.1% risk the maintenance margin should be set at  $277 \times \sqrt{2} \times 3.09 = 1,210$ .

For crude oil the standard deviation of daily changes is \$0.31 per barrel or \$310 per contract. For a 1% risk this means that the maintenance margin should be set at  $310 \times \sqrt{2} \times 2.33 = 1,021$ . For a 0.1% risk the maintenance margin should be set at  $310 \times \sqrt{2} \times 3.09 = 1,355$ .

NYMEX is interested in these types of calculations because it wants to set the maintenance margin level so that the balance in a trader's margin account has a very low probability of becoming negative. If a trader started with a balance just above the maintenance margin level and the market moved against her, there would be a margin call at the end of the first day and the trader would have until the end of the second day to meet the margin call. It is therefore the possibility of a large futures price movement over a two-day period that is of concern to NYMEX.

- (b) The initial margin is set at 1,362 for crude oil. (This is the maintenance margin divided by 0.75.) There are 151 margin calls and 7 times (out of 1201 days) where the investor is tempted to walk away. The initial margin is set at 1,215 for gold. There are 111 margin calls and 3 times (out of 826 days) when the investor is tempted to walk away. When the 0.1% risk level is used there are 3 times when the oil investor might walk away and 6 times when the gold investor might do so. These results suggest that

extreme movements occur more often than the normal distribution would suggest. Here are some notes on how I handled the Excel calculations. Suppose that the initial margin is in cell Q1 and the maintenance margin is in cell Q2. Suppose further that the change in the oil futures price is in column D of the spreadsheet and the margin balance is in column E. Consider cell E7. This is updated with an instruction of the form:

$$= \text{IF}(E6 < \$Q\$2, \$Q\$1, \text{IF}(E6 + D7 * 1000 > \$Q\$1, \$Q\$1, E6 + D7 * 1000))$$

Returning 1 in cell F7 if there has been a margin call and zero otherwise requires an instruction of the form:

$$= \text{IF}(E7 < \$Q\$2, 1, 0)$$

Returning 1 in cell G7 if there has been an incentive to walk away and zero otherwise requires an instruction of the form:

$$= \text{IF}(E6 + D7 * 1000 < 0, 1, 0)$$

# CHAPTER 3

## Hedging Strategies Using Futures

### Notes for the Instructor

This chapter discusses how long and short futures positions are used for hedging. It covers basis risk, hedge ratios, the use of stock index futures, and how to roll a hedge forward.

A number of people have pointed out a small inconsistency between the material in Chapter 3 and the CFA material in the previous edition. The issue is whether you base the number of contracts used for hedging on the futures price of the assets underlying a futures contract or the spot price of these assets. To be consistent with CFA, this edition does the former. The argument for doing so is that it is a way of adjusting for the marking to market of futures contracts. (See “tailing the hedge” material on page 58 and Problem 5.23 in Chapter 5.)

As will be evident from the slides, I cover the material in the chapter in the order in which it is presented. The section on arguments for and against hedging often generates a lively discussion. It is important to emphasize that the purpose of hedging is to reduce the standard deviation of the outcome, not to increase its expected value. I usually discuss Problem 3.17 at some stage to emphasize the point that, even in relatively simple situations, it is easy to make incorrect hedging decisions when you do not look at the big picture.

Business Snapshot 3.1 discusses hedging by gold mining companies. I use this to emphasize the importance of communicating with shareholders. I also like to discuss how investment banks hedge their risks when they enter into forward contracts with gold producers. (This is the second part of Business Snapshot 3.1.) I also like to ask students about the determinants of gold lease rates. If more gold producers choose to hedge, does the gold lease rate go up or down? (The answer is that it goes up because there is a greater demand on the part of investment banks for gold borrowing.)

Any of the Problems 3.23 to 3.26 can be used as assignment questions. My favorite is Problem 3.26.

### QUESTIONS AND PROBLEMS

#### Problem 3.1.

*Under what circumstances are (a) a short hedge and (b) a long hedge appropriate?*

A *short hedge* is appropriate when a company owns an asset and expects to sell that asset in the future. It can also be used when the company does not currently own the asset but expects to do so at some time in the future. A *long hedge* is appropriate when



a company knows it will have to purchase an asset in the future. It can also be used to offset the risk from an existing short position.

**Problem 3.2.**

*Explain what is meant by basis risk when futures contracts are used for hedging.*

*Basis risk* arises from the hedger's uncertainty as to the difference between the spot price and futures price at the expiration of the hedge.

**Problem 3.3.**

*Explain what is meant by a perfect hedge. Does a perfect hedge always lead to a better outcome than an imperfect hedge? Explain your answer.*

A *perfect hedge* is one that completely eliminates the hedger's risk. A perfect hedge does not always lead to a better outcome than an imperfect hedge. It just leads to a more certain outcome. Consider a company that hedges its exposure to the price of an asset. Suppose the asset's price movements prove to be favorable to the company. A perfect hedge totally neutralizes the company's gain from these favorable price movements. An imperfect hedge, which only partially neutralizes the gains, might well give a better outcome.

**Problem 3.4.**

*Under what circumstances does a minimum-variance hedge portfolio lead to no hedging at all?*

A minimum variance hedge leads to no hedging when the coefficient of correlation between the futures price changes and changes in the price of the asset being hedged is zero.

**Problem 3.5.**

*Give three reasons that the treasurer of a company might not hedge the company's exposure to a particular risk.*

(a) If the company's competitors are not hedging, the treasurer might feel that the company will experience less risk if it does not hedge. (See Table 3.1.) (b) The shareholders might not want the company to hedge. (c) If there is a loss on the hedge and a gain from the company's exposure to the underlying asset, the treasurer might feel that he or she will have difficulty justifying the hedging to other executives within the organization.

**Problem 3.6.**

*Suppose that the standard deviation of quarterly changes in the prices of a commodity is \$0.65, the standard deviation of quarterly changes in a futures price on the commodity is \$0.81, and the coefficient of correlation between the two changes is 0.8. What is the optimal hedge ratio for a three-month contract? What does it mean?*

The optimal hedge ratio is

$$0.8 \times \frac{0.65}{0.81} = 0.642$$

This means that the size of the futures position should be 64.2% of the size of the company's exposure in a three-month hedge.

**Problem 3.7.**

*A company has a \$20 million portfolio with a beta of 1.2. It would like to use futures contracts on the S&P 500 to hedge its risk. The index futures is currently standing at 1080, and each contract is for delivery of \$250 times the index. What is the hedge that minimizes risk? What should the company do if it wants to reduce the beta of the portfolio to 0.6?*

The formula for the number of contracts that should be shorted gives

$$1.2 \times \frac{20,000,000}{1080 \times 250} = 88.9$$

Rounding to the nearest whole number, 89 contracts should be shorted. To reduce the beta to 0.6, half of this position, or a short position in 44 contracts, is required.

**Problem 3.8.**

*In the Chicago Board of Trade's corn futures contract, the following delivery months are available: March, May, July, September, and December. State the contract that should be used for hedging when the expiration of the hedge is in*

- a. June
- b. July
- c. January

A good rule of thumb is to choose a futures contract that has a delivery month as close as possible to, but later than, the month containing the expiration of the hedge. The contracts that should be used are therefore (a) July, (b) September, and (c) March.

**Problem 3.9.**

*Does a perfect hedge always succeed in locking in the current spot price of an asset for a future transaction? Explain your answer.*

No. Consider, for example, the use of a forward contract to hedge a known cash inflow in a foreign currency. The forward contract locks in the forward exchange rate — which is in general different from the spot exchange rate.

**Problem 3.10.**

*Explain why a short hedger's position improves when the basis strengthens unexpectedly and worsens when the basis weakens unexpectedly.*

The basis is the amount by which the spot price exceeds the futures price. A short hedger is long the asset and short futures contracts. The value of his or her position therefore improves as the basis increases. Similarly it worsens as the basis decreases.

**Problem 3.11.**

Imagine you are the treasurer of a Japanese company exporting electronic equipment to the United States. Discuss how you would design a foreign exchange hedging strategy and the arguments you would use to sell the strategy to your fellow executives.

The simple answer to this question is that the treasurer should (a) estimate the company's future cash flows in Japanese yen and U.S. dollars and (b) enter into forward and futures contracts to lock in the exchange rate for the U.S. dollar cash flows.

However, this is not the whole story. As the gold jewelry example in Table 3.1 shows, the company should examine whether the magnitudes of the foreign cash flows depend on the exchange rate. For example, will the company be able to raise the price of its product in U.S. dollars if the yen appreciates? If the company can do so, its foreign exchange exposure may be quite low. The key estimates required are those showing the overall effect on the company's profitability of changes in the exchange rate at various times in the future. Once these estimates have been produced the company can choose between using futures and options to hedge its risk. The results of the analysis should be presented carefully to other executives. It should be explained that a hedge does not ensure that profits will be higher. It means that profit will be more certain. When futures/forwards are used both the downside and upside are eliminated. With options a premium is paid to eliminate only the downside.

**Problem 3.12.**

Suppose that in Example 3.2 of Section 3.3 the company decides to use a hedge ratio of 0.8. How does the decision affect the way in which the hedge is implemented and the result?

If the hedge ratio is 0.8, the company takes a long position in 16 NYM December oil futures contracts on June 8 when the futures price is \$68.00. It closes out its position on November 10. The spot price and futures price at this time are \$70.00 and \$69.10. The gain on the futures position is

$$(69.10 - 68.00) \times 16,000 = 17,600$$

The effective cost of the oil is therefore

$$20,000 \times 70 - 17,600 = 1,382,400$$

or \$69.12 per barrel. (This compares with \$68.90 per barrel when the company is fully hedged.)

**Problem 3.13.**

"If the minimum-variance hedge ratio is calculated as 1.0, the hedge must be perfect." Is this statement true? Explain your answer.

The statement is not true. The minimum variance hedge ratio is

$$\rho \frac{\sigma_S}{\sigma_F}$$

It is 1.0 when  $\rho = 0.5$  and  $\sigma_S = 2\sigma_F$ . Since  $\rho < 1.0$  the hedge is clearly not perfect.

**Problem 3.14.**

*“If there is no basis risk, the minimum variance hedge ratio is always 1.0.” Is this statement true? Explain your answer.*

The statement is true. Using the notation in the text, if the hedge ratio is 1.0, the hedger locks in a price of  $F_1 + b_2$ . Since both  $F_1$  and  $b_2$  are known this has a variance of zero and must be the best hedge.

**Problem 3.15.**

*“For an asset where futures prices are usually less than spot prices, long hedges are likely to be particularly attractive.” Explain this statement.*

A company that knows it will purchase a commodity in the future is able to lock in a price close to the futures price. This is likely to be particularly attractive when the futures price is less than the spot price.

**Problem 3.16.**

*The standard deviation of monthly changes in the spot price of live cattle is (in cents per pound) 1.2. The standard deviation of monthly changes in the futures price of live cattle for the closest contract is 1.4. The correlation between the futures price changes and the spot price changes is 0.7. It is now October 15. A beef producer is committed to purchasing 200,000 pounds of live cattle on November 15. The producer wants to use the December live-cattle futures contracts to hedge its risk. Each contract is for the delivery of 40,000 pounds of cattle. What strategy should the beef producer follow?*

The optimal hedge ratio is

$$0.7 \times \frac{1.2}{1.4} = 0.6$$

The beef producer requires a long position in  $200000 \times 0.6 = 120,000$  lbs of cattle. The beef producer should therefore take a long position in 3 December contracts closing out the position on November 15.

**Problem 3.17.**

*A corn farmer argues “I do not use futures contracts for hedging. My real risk is not the price of corn. It is that my whole crop gets wiped out by the weather.” Discuss this viewpoint. Should the farmer estimate his or her expected production of corn and hedge to try to lock in a price for expected production?*

Suppose that the weather is bad and the farmer’s production is lower than expected. Other farmers are likely to have been affected similarly. Corn production overall will be low and as a consequence the price of corn will be relatively high. The farmer is likely to be overhedged relative to actual production. The farmer’s problems arising from the bad harvest will be made worse by losses on the short futures position. This problem emphasizes the importance of looking at the big picture when hedging. The farmer is correct to question whether hedging price risk while ignoring other risks is a good strategy.

**Problem 3.18.**

On July 1, an investor holds 50,000 shares of a certain stock. The market price is \$30 per share. The investor is interested in hedging against movements in the market over the next month and decides to use the September Mini S&P 500 futures contract. The index futures price is currently 1,500 and one contract is for delivery of \$50 times the index. The beta of the stock is 1.3. What strategy should the investor follow?

A short position in

$$1.3 \times \frac{50,000 \times 30}{50 \times 1,500} = 26$$

contracts is required.

**Problem 3.19.**

Suppose that in Table 3.5 the company decides to use a hedge ratio of 1.5. How does the decision affect the way the hedge is implemented and the result?

If the company uses a hedge ratio of 1.5 in Table 3.5 it would at each stage short 150 contracts. The gain from the futures contracts would be

$$1.50 \times 1.70 = \$2.55 \text{ per barrel}$$

and the company would be \$0.85 per barrel better off.

**Problem 3.20.**

A futures contract is used for hedging. Explain why the marking to market of the contract can give rise to cash flow problems.

Suppose that you enter into a short futures contract to hedge the sale of an asset in six months. If the price of the asset rises sharply during the six months, the futures price will also rise and you may get margin calls. The margin calls will lead to cash outflows. Eventually the cash outflows will be offset by the extra amount you get when you sell the asset, but there is a mismatch in the timing of the cash outflows and inflows. Your cash outflows occur earlier than your cash inflows. A similar situation could arise if you used a long position in a futures contract to hedge the purchase of an asset and the asset's price fell sharply. An extreme example of what we are talking about here is provided by Metallgesellschaft (see Business Snapshot 3.2).

**Problem 3.21.**

An airline executive has argued: "There is no point in our using oil futures. There is just as much chance that the price of oil in the future will be less than the futures price as there is that it will be greater than this price." Discuss the executive's viewpoint.

It may well be true that there is just as much chance that the price of oil in the future will be above the futures price as that it will be below the futures price. This means that the use of a futures contract for speculation would be like betting on whether a coin comes up heads or tails. But it might make sense for the airline to use futures for hedging rather than speculation. The futures contract then has the effect of reducing risks. It can

be argued that an airline should not expose its shareholders to risks associated with the future price of oil when there are contracts available to hedge the risks.

**Problem 3.22.**

Suppose the one-year gold lease rate is 1.5% and the one-year risk-free rate is 5.0%. Both rates are compounded annually. Use the discussion in Business Snapshot 3.1 to calculate the maximum one-year forward price Goldman Sachs should quote for gold when the spot price is \$600.

Goldman Sachs can borrow 1 ounce of gold and sell it for \$600. It invests the \$600 at 5% so that it becomes \$630 at the end of the year. It must pay the lease rate of 1.5% on \$600. This is \$9 and leaves it with \$621. It follows that if it agrees to buy the gold for less than \$621 in one year it will make a profit.

**ASSIGNMENT QUESTIONS**

**Problem 3.23.**

The following table gives data on monthly changes in the spot price and the futures price for a certain commodity. Use the data to calculate a minimum variance hedge ratio.

Spot Price Change	+0.50	+0.61	-0.22	-0.35	+0.79
Futures Price Change	+0.56	+0.63	-0.12	-0.44	+0.60
Spot Price Change	+0.04	+0.15	+0.70	-0.51	-0.41
Futures price change	-0.06	+0.01	+0.80	-0.56	-0.46

Denote  $x_i$  and  $y_i$  by the  $i$ -th observation on the change in the futures price and the change in the spot price respectively.

$$\sum x_i = 0.96 \quad \sum y_i = 1.30$$

$$\sum x_i^2 = 2.4474 \quad \sum y_i^2 = 2.3594$$

$$\sum x_i y_i = 2.352$$

An estimate of  $\sigma_F$  is

$$\sqrt{\frac{2.4474}{9} - \frac{0.96^2}{10 \times 9}} = 0.5116$$

An estimate of  $\sigma_S$  is

$$\sqrt{\frac{2.3594}{9} - \frac{1.30^2}{10 \times 9}} = 0.4933$$

An estimate of  $\rho$  is

$$\frac{10 \times 2.352 - 0.96 \times 1.30}{\sqrt{(10 \times 2.4474 - 0.96^2)(10 \times 2.3594 - 1.30^2)}} = 0.981$$

The minimum variance hedge ratio is

$$\rho \frac{\sigma_S}{\sigma_F} = 0.981 \times \frac{0.4933}{0.5116} = 0.946$$

**Problem 3.24.**

It is July 16. A company has a portfolio of stocks worth \$100 million. The beta of the portfolio is 1.2. The company would like to use the CME December futures contract on the S&P 500 to change the beta of the portfolio to 0.5 during the period July 16 to November 16. The index futures price is currently 1,000, and each contract is on \$250 times the index.

- a. What position should the company take?
- b. Suppose that the company changes its mind and decides to increase the beta of the portfolio from 1.2 to 1.5. What position in futures contracts should it take?

(a) The company should short

$$\frac{(1.2 - 0.5) \times 100,000,000}{1000 \times 250}$$

or 280 contracts.

(b) The company should take a long position in

$$\frac{(1.5 - 1.2) \times 100,000,000}{1000 \times 250}$$

or 120 contracts.

**Problem 3.25.**

A fund manager has a portfolio worth \$50 million with a beta of 0.87. The manager is concerned about the performance of the market over the next two months and plans to use three-month futures contracts on the S&P 500 to hedge the risk. The current level of the index is 1250, one contract is on 250 times the index, the risk-free rate is 6% per annum, and the dividend yield on the index is 3% per annum. The current 3 month futures price is 1259.

- a. What position should the fund manager take to eliminate all exposure to the market over the next two months?
- b. Calculate the effect of your strategy on the fund manager's returns if the level of the market in two months is 1,000, 1,100, 1,200, 1,300, and 1,400. Assume that the one-month futures price is 0.25% higher than the index level at this time.

(a) The number of contracts the fund manager should short is

$$0.87 \times \frac{50,000,000}{1259 \times 250} = 138.20$$

Rounding to the nearest whole number, 138 contracts should be shorted.

(b) The following table shows that the strategy has the effect of locking in a return of close to \$490,000. To illustrate the calculations in the table consider the first column. If the index in two months is 1,000, the futures price is  $1000 \times 1.0025 = 1002.50$ . The gain on the short futures position is therefore

$$(1259 - 1002.50) \times 250 \times 138 = \$8,849,250$$

The return on the index is  $3 \times 2/12 = 0.5\%$  in the form of dividend and  $-250/1250 = -20\%$  in the form of capital gains. The total return on the index is therefore  $-19.5\%$ . The risk-free rate is  $1\%$  per two months. The return is therefore  $-20.5\%$  in excess of the risk-free rate. From the capital asset pricing model we expect the return on the portfolio to be  $0.87 \times -20.5\% = -17.835\%$  in excess of the risk-free rate. The portfolio return is therefore  $-16.835\%$ . The loss on the portfolio is  $0.16835 \times 50,000,000$  or \$8,417,500. When this is combined with the gain on the futures the total gain is \$431,750.

Index in Two months	1000	1100	1200	1300	1400
Futures Price (\$)	1002.50	1102.75	1203.00	1303.25	1403.50
Gain on Futures (\$)	8,849,250	5,390,625	1,932,000	-1,526,625	-4,985,250
Index Return	-19.5%	-11.5%	-3.5%	4.5%	12.5%
Excess Ind. Return	-20.5%	-12.5%	-4.5%	3.5%	11.5%
Excess Port. Return	-17.835%	-10.875%	-3.915%	3.045%	10.005%
Port. Return	-16.835%	-9.875%	-2.915%	4.045%	11.005%
Port. Gain (\$)	-8,417,500	-4,937,500	-1,457,500	2,022,500	5,502,500
Total Gain (\$)	431,750	453,125	488,500	495,875	517,250

### Problem 3.26.

*It is now October 2007. A company anticipates that it will purchase 1 million pounds of copper in each of February 2008, August 2008, February 2009, and August 2009. The company has decided to use the futures contracts traded in the COMEX division of the New York Mercantile Exchange to hedge its risk. One contract is for the delivery of 25,000 pounds of copper. The initial margin is \$2,000 per contract and the maintenance margin is \$1,500 per contract. The company's policy is to hedge 80% of its exposure. Contracts with maturities up to 13 months into the future are considered to have sufficient liquidity to meet the company's needs. Devise a hedging strategy for the company. Do not make the tailing adjustments described in Section 3.4.*



Assume the market prices (in cents per pound) today and at future dates are as follows. What is the impact of the strategy you propose on the price the company pays for copper? What is the initial margin requirement in October 2004? Is the company subject to any margin calls?

Date	Oct 2007	Feb 2008	Aug 2008	Feb 2009	Aug 2009
Spot Price	372.00	369.00	365.00	377.00	388.00
Mar 2008 Futures Price	372.30	369.10			
Sep 2008 Futures Price	372.80	370.20	364.80		
Mar 2009 Futures Price		370.70	364.30	376.70	
Sep 2009 Futures Price			364.20	376.50	388.20

What is the impact of the strategy you propose on the price the company pays for copper? What is the initial margin requirement in October 2007? Is the company subject to any margin calls?

To hedge the February 2008 purchase the company should take a long position in March 2008 contracts for the delivery of 800,000 pounds of copper. The total number of contracts required is  $800,000/25,000 = 32$ . Similarly a long position in 32 September 2008 contracts is required to hedge the August 2008 purchase. For the February 2009 purchase the company could take a long position in 32 September 2008 contracts and roll them into March 2009 contracts during August 2008. (As an alternative, the company could hedge the February 2009 purchase by taking a long position in 32 March 2008 contracts and rolling them into March 2009 contracts.) For the August 2009 purchase the company could take a long position in 32 September 2008 and roll them into September 2009 contracts during August 2008.

The strategy is therefore as follows

- Oct. 2007: Enter into long position in 96 Sept. 2008 contracts  
Enter into a long position in 32 Mar. 2008 contracts
- Feb. 2008: Close out 32 Mar. 2008 contracts
- Aug. 2008: Close out 96 Sept. 2008 contracts  
Enter into long position in 32 Mar. 2009 contracts  
Enter into long position in 32 Sept. 2009 contracts
- Feb. 2009: Close out 32 Mar. 2009 contracts
- Aug. 2009: Close out 32 Sept. 2009 contracts

With the market prices shown the company pays

$$369.00 + 0.8 \times (372.30 - 369.10) = 371.56$$

for copper in February, 2008. It pays

$$365.00 + 0.8 \times (372.80 - 364.80) = 371.40$$

for copper in August 2008. As far as the February 2009 purchase is concerned, it loses  $372.80 - 364.80 = 8.00$  on the September 2008 futures and gains  $376.70 - 364.30 = 12.40$

on the February 2009 futures. The net price paid is therefore

$$377.00 + 0.8 \times 8.00 - 0.8 \times 12.40 = 373.48$$

As far as the August 2009 purchase is concerned, it loses  $372.80 - 364.80 = 8.00$  on the September 2008 futures and gains  $388.20 - 364.20 = 24.00$  on the September 2009 futures. The net price paid is therefore

$$388.00 + 0.8 \times 8.00 - 0.8 \times 24.00 = 375.20$$

The hedging scheme succeeds in keeping the price paid in the range 371.40 to 375.20.

In October 2007 the initial margin requirement on the 128 contracts is  $128 \times \$2,000$  or \$256,000. There is a margin call when the futures price drops by more than 2 cents. This happens to the March 2008 contract between October 2007 and February 2008, to the September 2008 contract between October 2007 and February 2008, and to the September 2008 contract between February 2008 and August 2008.

# CHAPTER 4

## Interest Rates

### Notes for the Instructor

This chapter together with Chapters 6 and 7 emphasizes that, for a derivatives trader, risk-free rates are the rates derived from LIBOR markets, Eurodollar futures, and swap markets. The reasons why derivatives traders do not use Treasury rates as risk-free rates are outlined in Business Snapshot 4.1. Chapter 7 continues this discussion by explaining that swap rates have very little credit risk because a bank can earn the swap rate by making a series of short term loans to AA-rated companies.

A new feature of this chapter is the expanded treatment of liquidity preference theory in Section 4.10.

I like to spend a some time explaining compounding frequency issues. I make it clear to students that we are talking about nothing more than a unit of measurement for interest rates. Moving from quarterly compounding to continuous compounding is like changing the unit of measurement of distance from miles to kilometers. When students are introduced to continuous compounding early in a course, I find they have very little difficulty with it.

The first part of the chapter discusses zero rates, bond valuation, bond yields, par yields, and the calculation of the Treasury zero curve. The slides mirror the examples in the text. When covering the bootstrap method to calculate the Treasury zero curve, I mention that Chapter 7 explains how the same procedure can be used to calculate the LIBOR/swap zero curve. I also point out that the bootstrap method is a very popular approach, but it is not the only one that is used in practice. For example, some analysts use cubic or exponential splines.

I spend some time on the relationship between spot and forward interest rates and combine this with a discussion of FRAs and theories of the term structure. I explain that it is possible to enter into transactions that lock in the forward rate for a future time period and then discuss the Orange County story (Business Snapshot 4.2). Orange County entered into contracts (often highly levered) that paid off if the forward rate was higher than the realized future spot rate (An example of such a contract is an FRA where fixed is received and floating is paid). This worked well in 1992 and 1993, but led to a huge loss in 1994.

Sections 4.8 and 4.9 cover duration and convexity. Duration is a widely used concept in derivatives markets. The chapter explains that the  $\Delta B/B = -D\Delta y$  relationship holds when rates are continuously compounded. When some other compounding frequency is used the same relationship is true provided  $D$  is defined as the modified duration. I like to illustrate the truth of the duration relationship with numerical example similar to those in the text.

Problems 4.24 to 4.28 can be used as assignment questions. My favorites are 4.27 and 4.28.

## QUESTIONS AND PROBLEMS

### Problem 4.1.

A bank quotes you an interest rate of 14% per annum with quarterly compounding. What is the equivalent rate with (a) continuous compounding and (b) annual compounding?

(a) The rate with continuous compounding is

$$4 \ln \left( 1 + \frac{0.14}{4} \right) = 0.1376$$

or 13.76% per annum.

(b) The rate with annual compounding is

$$\left( 1 + \frac{0.14}{4} \right)^4 - 1 = 0.1475$$

or 14.75% per annum.

### Problem 4.2.

What is meant by LIBOR and LIBID. Which is higher?

LIBOR is the London InterBank Offered Rate. It is the rate a bank quotes for deposits it is prepared to place with other banks. LIBID is the London InterBank Bid rate. It is the rate a bank quotes for deposits from other banks. LIBOR is greater than LIBID.

### Problem 4.3.

The six-month and one-year zero rates are both 10% per annum. For a bond that has a life of 18 months and pays a coupon of 8% per annum (with semiannual payments and one having just been made), the yield is 10.4% per annum. What is the bond's price? What is the 18-month zero rate? All rates are quoted with semiannual compounding.

Suppose the bond has a face value of \$100. Its price is obtained by discounting the cash flows at 10.4%. The price is

$$\frac{4}{1.052} + \frac{4}{1.052^2} + \frac{104}{1.052^3} = 96.74$$

If the 18-month zero rate is  $R$ , we must have

$$\frac{4}{1.05} + \frac{4}{1.05^2} + \frac{104}{(1 + R/2)^3} = 96.74$$

which gives  $R = 10.42\%$ .

**Problem 4.4.**

An investor receives \$1,100 in one year in return for an investment of \$1,000 now. Calculate the percentage return per annum with a) Annual compounding, b) Semiannual compounding, c) Monthly compounding and d) Continuous compounding.

(a) With annual compounding the return is

$$\frac{1100}{1000} - 1 = 0.1$$

or 10% per annum.

(b) With semi-annual compounding the return is  $R$  where

$$1000 \left(1 + \frac{R}{2}\right)^2 = 1100$$

i.e.,

$$1 + \frac{R}{2} = \sqrt{1.1} = 1.0488$$

so that  $R = 0.0976$ . The percentage return is therefore 9.76% per annum.

(c) With monthly compounding the return is  $R$  where

$$1000 \left(1 + \frac{R}{12}\right)^{12} = 1100$$

i.e.,

$$\left(1 + \frac{R}{12}\right) = \sqrt[12]{1.1} = 1.00797$$

so that  $R = 0.0957$ . The percentage return is therefore 9.57% per annum.

(d) With continuous compounding the return is  $R$  where:

$$1000e^R = 1100$$

i.e.,

$$e^R = 1.1$$

so that  $R = \ln 1.1 = 0.0953$ . The percentage return is therefore 9.53% per annum.

**Problem 4.5.**

Suppose that zero interest rates with continuous compounding are as follows:

Maturity (months)	Rate (% per annum)
3	8.0
6	8.2
9	8.4
12	8.5
15	8.6
18	8.7

Calculate forward interest rates for the second, third, fourth, fifth, and sixth quarters.

The forward rates with continuous compounding are as follows

Qtr 2:	8.4%
Qtr 3:	8.8%
Qtr 4:	8.8%
Qtr 5:	9.0%
Qtr 6:	9.2%

**Problem 4.6.**

Assuming that zero rates are as in Problem 4.5, what is the value of an FRA that enables the holder to earn 9.5% for a three-month period starting in one year on a principal of \$1,000,000? The interest rate is expressed with quarterly compounding.

The forward rate is 9.0% with continuous compounding or 9.102% with quarterly compounding. From equation (4.9), the value of the FRA is therefore

$$[1,000,000 \times 0.25 \times (0.095 - 0.09102)]e^{-0.086 \times 1.25} = 893.56$$

or \$893.56.

**Problem 4.7.**

The term structure of interest rates is upward sloping. Put the following in order of magnitude:

- The five-year zero rate
- The yield on a five-year coupon-bearing bond
- The forward rate corresponding to the period between 4.75 and 5 years in the future

What is the answer to this question when the term structure of interest rates is downward sloping?

When the term structure is upward sloping,  $c > a > b$ . When it is downward sloping,  $b > a > c$ .

**Problem 4.8.**

What does duration tell you about the sensitivity of a bond portfolio to interest rates. What are the limitations of the duration measure?

Duration provides information about the effect of a small parallel shift in the yield curve on the value of a bond portfolio. The percentage decrease in the value of the portfolio equals the duration of the portfolio multiplied by the amount by which interest rates are increased in the small parallel shift. The duration measure has the following limitation. It applies only to parallel shifts in the yield curve that are small.

**Problem 4.9.**

*What rate of interest with continuous compounding is equivalent to 15% per annum with monthly compounding?*

The rate of interest is  $R$  where:

$$e^R = \left(1 + \frac{0.15}{12}\right)^{12}$$

i.e.,

$$\begin{aligned} R &= 12 \ln \left(1 + \frac{0.15}{12}\right) \\ &= 0.1491 \end{aligned}$$

The rate of interest is therefore 14.91% per annum.

**Problem 4.10.**

*A deposit account pays 12% per annum with continuous compounding, but interest is actually paid quarterly. How much interest will be paid each quarter on a \$10,000 deposit?*

The equivalent rate of interest with quarterly compounding is  $R$  where

$$e^{0.12} = \left(1 + \frac{R}{4}\right)^4$$

or

$$R = 4(e^{0.03} - 1) = 0.1218$$

The amount of interest paid each quarter is therefore:

$$10,000 \times \frac{0.1218}{4} = 304.55$$

or \$304.55.

**Problem 4.11.**

*Suppose that 6-month, 12-month, 18-month, 24-month, and 30-month zero rates are 4%, 4.2%, 4.4%, 4.6%, and 4.8% per annum with continuous compounding respectively. Estimate the cash price of a bond with a face value of 100 that will mature in 30 months pays a coupon of 4% per annum semiannually.*

The bond pays \$2 in 6, 12, 18, and 24 months, and \$102 in 30 months. The cash price is

$$2e^{-0.04 \times 0.5} + 2e^{-0.042 \times 1.0} + 2e^{-0.044 \times 1.5} + 2e^{-0.046 \times 2} + 102e^{-0.048 \times 2.5} = 98.04$$

**Problem 4.12.**

A three-year bond provides a coupon of 8% semiannually and has a cash price of 104. What is the bond's yield?

The bond pays \$4 in 6, 12, 18, 24, and 30 months, and \$104 in 36 months. The bond yield is the value of  $y$  that solves

$$4e^{-0.5y} + 4e^{-1.0y} + 4e^{-1.5y} + 4e^{-2.0y} + 4e^{-2.5y} + 104e^{-3.0y} = 104$$

Using the *Goal Seek* tool in Excel  $y = 0.06407$  or 6.407%.

**Problem 4.13.**

Suppose that the 6-month, 12-month, 18-month, and 24-month zero rates are 5%, 6%, 6.5%, and 7% respectively. What is the two-year par yield?

Using the notation in the text,  $m = 2$ ,  $d = e^{-0.07 \times 2} = 0.8694$ . Also

$$A = e^{-0.05 \times 0.5} + e^{-0.06 \times 1.0} + e^{-0.065 \times 1.5} + e^{-0.07 \times 2.0} = 3.6935$$

The formula in the text gives the par yield as

$$\frac{(100 - 100 \times 0.8694) \times 2}{3.6935} = 7.072$$

To verify that this is correct we calculate the value of a bond that pays a coupon of 7.072% per year (that is 3.5365 every six months). The value is

$$3.536e^{-0.05 \times 0.5} + 3.5365e^{-0.06 \times 1.0} + 3.536e^{-0.065 \times 1.5} + 103.536e^{-0.07 \times 2.0} = 100$$

verifying that 7.072% is the par yield.

**Problem 4.14.**

Suppose that zero interest rates with continuous compounding are as follows:

Maturity (years)	Rate (% per annum)
1	2.0
2	3.0
3	3.7
4	4.2
5	4.5



Calculate forward interest rates for the second, third, fourth, and fifth years.

The forward rates with continuous compounding are as follows:

Year 2:	4.0%
Year 3:	5.1%
Year 4:	5.7%
Year 5:	5.7%

**Problem 4.15.**

Use the rates in Problem 4.14 to value an FRA where you will pay 5% for the third year on \$1 million.

The forward rate is 5.1% with continuous compounding or  $e^{0.051 \times 1} - 1 = 5.232\%$  with annual compounding. The 3-year interest rate is 3.7% with continuous compounding. From equation (4.10), the value of the FRA is therefore

$$[1,000,000 \times (0.05232 - 0.05) \times 1]e^{-0.037 \times 3} = 2,078.85$$

or \$2,078.85.

**Problem 4.16.**

A 10-year, 8% coupon bond currently sells for \$90. A 10-year, 4% coupon bond currently sells for \$80. What is the 10-year zero rate? (Hint: Consider taking a long position in two of the 4% coupon bonds and a short position in one of the 8% coupon bonds.)

Taking a long position in two of the 4% coupon bonds and a short position in one of the 8% coupon bonds leads to the following cash flows

$$\begin{aligned}\text{Year 0 : } & 90 - 2 \times 80 = -70 \\ \text{Year 10 : } & 200 - 100 = 100\end{aligned}$$

because the coupons cancel out. \$100 in 10 years time is equivalent to \$70 today. The 10-year rate,  $R$ , (continuously compounded) is therefore given by

$$100 = 70e^{10R}$$

The rate is

$$\frac{1}{10} \ln \frac{100}{70} = 0.0357$$

or 3.57% per annum.

**Problem 4.17.**

Explain carefully why liquidity preference theory is consistent with the observation that the term structure of interest rates tends to be upward sloping more often than it is downward sloping.

If long-term rates were simply a reflection of expected future short-term rates, we would expect the term structure to be downward sloping as often as it is upward sloping. (This is based on the assumption that half of the time investors expect rates to increase and half of the time investors expect rates to decrease). Liquidity preference theory argues that long term rates are high relative to expected future short-term rates. This means that the term structure should be upward sloping more often than it is downward sloping.

**Problem 4.18.**

*“When the zero curve is upward sloping, the zero rate for a particular maturity is greater than the par yield for that maturity. When the zero curve is downward sloping the reverse is true.” Explain why this is so.*

The par yield is the yield on a coupon-bearing bond. The zero rate is the yield on a zero-coupon bond. When the yield curve is upward sloping, the yield on an  $N$ -year coupon-bearing bond is less than the yield on an  $N$ -year zero-coupon bond. This is because the coupons are discounted at a lower rate than the  $N$ -year rate and drag the yield down below this rate. Similarly, when the yield curve is downward sloping, the yield on an  $N$ -year coupon bearing bond is higher than the yield on an  $N$ -year zero-coupon bond.

**Problem 4.19.**

*Why are U.S. Treasury rates significantly lower than other rates that are close to risk free?*

There are three reasons (see Business Snapshot 4.1).

(i) Treasury bills and Treasury bonds must be purchased by financial institutions to fulfill a variety of regulatory requirements. This increases demand for these Treasury instruments driving the price up and the yield down.

(ii) The amount of capital a bank is required to hold to support an investment in Treasury bills and bonds is substantially smaller than the capital required to support a similar investment in other very-low-risk instruments.

(iii) In the United States, Treasury instruments are given a favorable tax treatment compared with most other fixed-income investments because they are not taxed at the state level.

**Problem 4.20.**

*Why does a loan in the repo market involve very little credit risk?*

A repo is a contract where an investment dealer who owns securities agrees to sell them to another company now and buy them back later at a slightly higher price. The other company is providing a loan to the investment dealer. This loan involves very little credit risk. If the borrower does not honor the agreement, the lending company simply keeps the securities. If the lending company does not keep to its side of the agreement, the original owner of the securities keeps the cash.

**Problem 4.21.**

*Explain why an FRA is equivalent to the exchange of a floating rate of interest for a fixed rate of interest.*

A FRA is an agreement that a certain specified interest rate,  $R_K$ , will apply to a certain principal,  $L$ , for a certain specified future time period. Suppose that the rate observed in the market for the future time period at the beginning of the time period proves to be  $R_M$ . If the FRA is an agreement that  $R_K$  will apply when the principal is invested, the holder of the FRA can borrow the principal at  $R_M$  and then invest it at  $R_K$ . The net cash flow at the end of the period is then an inflow of  $R_K L$  and an outflow of  $R_M L$ . If the FRA is an agreement that  $R_K$  will apply when the principal is borrowed, the holder of the FRA can invest the borrowed principal at  $R_M$ . The net cash flow at the end of the period is then an inflow of  $R_M L$  and an outflow of  $R_K L$ . In either case we see that the FRA involves the exchange of a fixed rate of interest,  $R_K$ , on the principal of  $L$  for the floating rate of interest observed in the market,  $R_M$ .

**Problem 4.22.**

A five-year bond with a yield of 11% (continuously compounded) pays an 8% coupon at the end of each year.

- a. What is the bond's price?
- b. What is the bond's duration?
- c. Use the duration to calculate the effect on the bond's price of a 0.2% decrease in its yield.
- d. Recalculate the bond's price on the basis of a 10.8% per annum yield and verify that the result is in agreement with your answer to (c).

(a) The bond's price is

$$8e^{-0.11} + 8e^{-0.11 \times 2} + 8e^{-0.11 \times 3} + 8e^{-0.11 \times 4} + 108e^{-0.11 \times 5} = 86.80$$

(b) The bond's duration is

$$\frac{1}{86.80} [8e^{-0.11} + 2 \times 8e^{-0.11 \times 2} + 3 \times 8e^{-0.11 \times 3} + 4 \times 8e^{-0.11 \times 4} + 5 \times 108e^{-0.11 \times 5}]$$

$$= 4.256 \text{ years}$$

(c) Since, with the notation in the chapter

$$\Delta B = -BD\Delta y$$

the effect on the bond's price of a 0.2% decrease in its yield is

$$86.80 \times 4.256 \times 0.002 = 0.74$$

The bond's price should increase from 86.80 to 87.54.

(d) With a 10.8% yield the bond's price is

$$8e^{-0.108} + 8e^{-0.108 \times 2} + 8e^{-0.108 \times 3} + 8e^{-0.108 \times 4} + 108e^{-0.108 \times 5} = 87.54$$

This is consistent with the answer in (c).

**Problem 4.23.**

The cash prices of six-month and one-year Treasury bills are 94.0 and 89.0. A 1.5-year bond that will pay coupons of \$4 every six months currently sells for \$94.84. A two-year bond that will pay coupons of \$5 every six months currently sells for \$97.12. Calculate the six-month, one-year, 1.5-year, and two-year zero rates.

The 6-month rate (with continuous compounding) is  $2\ln(1 + 6/94) = 12.38\%$ . The 12-month rate is  $\ln(1 + 11/89) = 11.65\%$ .

For the 1.5-year bond we must have

$$4e^{-0.1238 \times 0.5} + 4e^{-0.1165 \times 1.0} + 104e^{-1.5R} = 94.84$$

where  $R$  is the 1.5-year spot rate. It follows that

$$3.76 + 3.56 + 104e^{-1.5R} = 94.84$$

$$e^{-1.5R} = 0.8415$$

$$R = 0.115$$

or 11.5%. For the 2-year bond we must have

$$5e^{-0.1238 \times 0.5} + 5e^{-0.1165 \times 1.0} + 5e^{-0.115 \times 1.5} + 105e^{-2R} = 97.12$$

where  $R$  is the 2-year spot rate. It follows that

$$e^{-2R} = 0.7977$$

$$R = 0.113$$

or 11.3%.

## ASSIGNMENT QUESTIONS

**Problem 4.24.**

An interest rate is quoted as 5% per annum with semiannual compounding. What is the equivalent rate with (a) annual compounding, (b) monthly compounding, and (c) continuous compounding.

(a) With annual compounding the rate is  $1.025^2 - 1 = 0.050625$  or 5.0625%

(b) With monthly compounding the rate is  $12 \times (1.025^{1/6} - 1) = 0.04949$  or 4.949%.

(c) With continuous compounding the rate is  $2 \times \ln 1.025 = 0.04939$  or 4.939%.

**Problem 4.25.**

The 6-month, 12-month, 18-month, and 24-month zero rates are 4%, 4.5%, 4.75%, and 5% with semiannual compounding.

- (a) What are the rates with continuous compounding?  
 (b) What is the forward rate for the six-month period beginning in 18 months  
 (c) What is the value of an FRA that promises to pay you 6% (compounded semiannually) on a principal of \$1 million for the six-month period starting in 18 months?

(a) With continuous compounding the 6-month rate is  $2 \ln 1.02 = 0.039605$  or 3.961%. The 12-month rate is  $2 \ln 1.0225 = 0.044501$  or 4.4501%. The 18-month rate is  $2 \ln 1.02375 = 0.046945$  or 4.6945%. The 24-month rate is  $2 \ln 1.025 = 0.049385$  or 4.9385%.

(b) The forward rate (expressed with continuous compounding) is from equation (4.5)

$$\frac{4.9385 \times 2 - 4.6945 \times 1.5}{0.5}$$

or 5.6707%. When expressed with semiannual compounding this is  $2(e^{0.056707 \times 0.5} - 1) = 0.057518$  or 5.7518%.

(c) The value of an FRA that where you will receive 6% for the six month period starting in 18 months is from equation (4.9)

$$1,000,000 \times (0.06 - 0.057518) \times 0.5 e^{-0.049385 \times 2} = 1,124$$

or \$1,124.

**Problem 4.26.**

What is the two-year par yield when the zero rates are as in Problem 4.25? What is the yield on a two-year bond that pays a coupon equal to the par yield?

The value,  $A$  of an annuity paying off \$1 every six months is

$$e^{-0.039605 \times 0.5} + e^{-0.044501 \times 1} + e^{-0.046945 \times 1.5} + e^{-0.049385 \times 2} = 3.7748$$

The present value of \$1 received in two years,  $d$ , is  $e^{-0.049385 \times 2} = 0.90595$ . From the formula in Section 4.4 the par yield is

$$\frac{(100 - 100 \times 0.90595) \times 2}{3.7748} = 4.983$$

or 4.983%.

**Problem 4.27.**

The following table gives the prices of bonds

Bond Principal (\$)	Time to Maturity (years)	Annual Coupon (\$)*	Bond Price (\$)
100	0.50	0.0	98
100	1.00	0.0	95
100	1.50	6.2	101
100	2.00	8.0	104

\* Half the stated coupon is assumed to be paid every six months.

- Calculate zero rates for maturities of 6 months, 12 months, 18 months, and 24 months.
- What are the forward rates for the periods: 6 months to 12 months, 12 months to 18 months, 18 months to 24 months?
- What are the 6-month, 12-month, 18-month, and 24-month par yields for bonds that provide semiannual coupon payments?
- Estimate the price and yield of a two-year bond providing a semiannual coupon of 7% per annum.

- (a) The zero rate for a maturity of six months, expressed with continuous compounding is  $2 \ln(1 + 2/98) = 4.0405\%$ . The zero rate for a maturity of one year, expressed with continuous compounding is  $\ln(1 + 5/95) = 5.1293$ . The 1.5-year rate is  $R$  where

$$3.1e^{-0.040405 \times 0.5} + 3.1e^{-0.051293 \times 1} + 103.1e^{-R \times 1.5} = 101$$

The solution to this equation is  $R = 0.054429$ . The 2.0-year rate is  $R$  where

$$4e^{-0.040405 \times 0.5} + 4e^{-0.051293 \times 1} + 4e^{-0.054429 \times 1.5} + 104e^{-R \times 2} = 104$$

The solution to this equation is  $R = 0.058085$ . These results are shown in the table below

Maturity (years)	Zero Rate (%)	Forward Rate (%)	Par Yield semi ann. (%)	Par Yield cont. comp.
0.5	4.0405	4.0405	4.0816	4.0405
1.0	5.1293	6.2181	5.1813	5.1154
1.5	5.4429	6.0700	5.4986	5.4244
2.0	5.8085	6.9054	5.8620	5.7778

- The continuously compounded forward rates calculated using equation (4.5) are shown in the third column of the table
- The par yield, expressed with semiannual compounding, can be calculated from the formula in Section 4.4. It is shown in the fourth column of the table. In the fifth column of the table it is converted to continuous compounding

(d) The price of the bond is

$$3.5e^{-0.040405 \times 0.5} + 3.5e^{-0.051293 \times 1} + 3.5e^{-0.054429 \times 1.5} + 103.5e^{-0.058085 \times 2} = 102.13$$

The yield on the bond,  $y$  satisfies

$$3.5e^{-y \times 0.5} + 3.5e^{-y \times 1.0} + 3.5e^{-y \times 1.5} + 103.5e^{-y \times 2.0} = 102.13$$

The solution to this equation is  $y = 0.057723$ . The bond yield is therefore 5.7723%.

**Problem 4.28.**

*Portfolio A consists of a one-year zero-coupon bond with a face value of \$2,000 and a 10-year zero-coupon bond with a face value of \$6,000. Portfolio B consists of a 5.95-year zero-coupon bond with a face value of \$5,000. The current yield on all bonds is 10% per annum.*

a. *Show that both portfolios have the same duration.*

b. *Show that the percentage changes in the values of the two portfolios for a 0.1% per annum increase in yields are the same.*

c. *What are the percentage changes in the values of the two portfolios for a 5% per annum increase in yields?*

(a) The duration of Portfolio A is

$$\frac{1 \times 2000e^{-0.1 \times 1} + 10 \times 6000e^{-0.1 \times 10}}{2000e^{-0.1 \times 1} + 6000e^{-0.1 \times 10}} = 5.95$$

Since this is also the duration of Portfolio B, the two portfolios do have the same duration.

(b) The value of Portfolio A is

$$2000e^{-0.1} + 6000e^{-0.1 \times 10} = 4016.95$$

When yields increase by 10 basis points its value becomes

$$2000e^{-0.101} + 6000e^{-0.101 \times 10} = 3993.18$$

The percentage decrease in value is

$$\frac{23.77 \times 100}{4016.95} = 0.59\%$$

The value of Portfolio B is

$$5000e^{-0.1 \times 5.95} = 2757.81$$

When yields increase by 10 basis points its value becomes

$$5000e^{-0.101 \times 5.95} = 2741.45$$

The percentage decrease in value is

$$\frac{16.36 \times 100}{2757.81} = 0.59\%$$

The percentage changes in the values of the two portfolios for a 10 basis point increase in yields are therefore the same.

(c) When yields increase by 5% the value of Portfolio A becomes

$$2000e^{-0.15} + 6000e^{-0.15 \times 10} = 3060.20$$

and the value of Portfolio B becomes

$$5000e^{-0.15 \times 5.95} = 2048.15$$

The percentage reduction in the values of the two portfolios are:

$$\text{Portfolio A: } \frac{956.75}{4016.95} \times 100 = 23.82$$

$$\text{Portfolio B: } \frac{709.66}{2757.81} \times 100 = 25.73$$

Since the percentage decline in value of Portfolio A is less than that of Portfolio B, Portfolio A has a greater convexity (see Figure 4.2 in text).



# CHAPTER 5

## Determination of Forward and Futures Prices

### Notes for the Instructor

This chapter covers the relationship between forward/futures prices and spot prices. The approach used in the chapter is to produce results for forward prices first and then argue that futures prices are very close to forward prices. The early part of the chapter explains short selling and the difference between investment and consumption assets.

I usually go through the material in Section 5.4 fairly carefully to make sure that students understand the nature of the arguments that are used. (Business Snapshot 5.1, a description of Joseph Jett's trading at Kidder Peabody, helps to explain why equation 5.1 holds.) I then go through Sections 5.5 and 5.6 fairly quickly because the arguments in those sections are really just extensions of the argument in Section 5.4. However, it is necessary to explain carefully the difference between a known cash dividend and a known dividend yield.

When covering Section 5.7, I emphasize the distinction between  $f$  (the value of a long forward contract) and  $F_0$  (the forward price). This often causes confusion. I like to go through Business Snapshot 5.2 to help students understand the issue.

If time permits I like to go through the material in the appendix. It reinforces the students' understanding of how futures contracts work and provides an interesting pure arbitrage argument.

The material in Sections 5.9 and 5.10 follows naturally from the material in Sections 5.4 to 5.6. I try to illustrate all of the formulas with numerical examples taken from current market quotes. The interpretation of a foreign currency as an investment providing a yield equal to the foreign risk-free rate needs to be explained carefully. I also like to spend some time discussing the fact that the variable underlying the CME Nikkei futures contract is not something that can be traded (see Business Snapshot 5.3).

It is important that students understand the distinction between assets that are held solely for investment by a significant number of investors and those that are not. This distinction is made right at the beginning of the chapter.

Section 5.14 ties the relationship between a futures prices and an expected future spot price to the notion of systematic risk, which will probably be familiar to students from other courses they have taken.

Problems 5.26 and 5.28 can be used for discussion in class. Problems 5.24, 5.25, and 5.27 can be used as assignment questions. My favorites are 5.25 and 5.27.

## QUESTIONS AND PROBLEMS

### Problem 5.1.

*Explain what happens when an investor shorts a certain share.*

The investor's broker borrows the shares from another client's account and sells them in the usual way. To close out the position, the investor must purchase the shares. The broker then replaces them in the account of the client from whom they were borrowed. The party with the short position must remit to the broker dividends and other income paid on the shares. The broker transfers these funds to the account of the client from whom the shares were borrowed. Occasionally the broker runs out of places from which to borrow the shares. The investor is then short squeezed and has to close out the position immediately.

### Problem 5.2.

*What is the difference between the forward price and the value of a forward contract?*

The forward price of an asset today is the price at which you would agree to buy or sell the asset at a future time. The value of a forward contract is zero when you first enter into it. As time passes the underlying asset price changes and the value of the contract may become positive or negative.

### Problem 5.3.

*Suppose that you enter into a six-month forward contract on a non-dividend-paying stock when the stock price is \$30 and the risk-free interest rate (with continuous compounding) is 12% per annum. What is the forward price?*

The forward price is

$$30e^{0.12 \times 0.5} = \$31.86$$

### Problem 5.4.

*A stock index currently stands at 350. The risk-free interest rate is 8% per annum (with continuous compounding) and the dividend yield on the index is 4% per annum. What should the futures price for a four-month contract be?*

The futures price is

$$350e^{(0.08 - 0.04) \times 0.3333} = \$354.7$$

### Problem 5.5.

*Explain carefully why the futures price of gold can be calculated from its spot price and other observable variables whereas the futures price of copper cannot.*

Gold is an investment asset. If the futures price is too high, investors will find it profitable to increase their holdings of gold and short futures contracts. If the futures price is too low, they will find it profitable to decrease their holdings of gold and go long in the futures market. Copper is a consumption asset. If the futures price is too high,

a strategy of buy copper and short futures works. However, because investors do not in general hold the asset, the strategy of sell copper and buy futures is not available to them. There is therefore an upper bound, but no lower bound, to the futures price.

**Problem 5.6.**

*Explain carefully the meaning of the terms convenience yield and cost of carry. What is the relationship between futures price, spot price, convenience yield, and cost of carry?*

*Convenience yield* measures the extent to which there are benefits obtained from ownership of the physical asset that are not obtained by owners of long futures contracts. The *cost of carry* is the interest cost plus storage cost less the income earned. The futures price,  $F_0$ , and spot price,  $S_0$ , are related by

$$F_0 = S_0 e^{(c-y)T}$$

where  $c$  is the cost of carry,  $y$  is the convenience yield, and  $T$  is the time to maturity of the futures contract.

**Problem 5.7.**

*Explain why a foreign currency can be treated as an asset providing a known yield.*

A foreign currency provides a known interest rate, but the interest is received in the foreign currency. The value in the domestic currency of the income provided by the foreign currency is therefore known as a percentage of the value of the foreign currency. This means that the income has the properties of a known yield.

**Problem 5.8.**

*Is the futures price of a stock index greater than or less than the expected future value of the index? Explain your answer.*

The futures price of a stock index is always less than the expected future value of the index. This follows from Section 5.14 and the fact that the index has positive systematic risk. For an alternative argument, let  $\mu$  be the expected return required by investors on the index so that  $E(S_T) = S_0 e^{(\mu-q)T}$ . Because  $\mu > r$  and  $F_0 = S_0 e^{(r-q)T}$ , it follows that  $E(S_T) > F_0$ .

**Problem 5.9.**

*A one-year long forward contract on a non-dividend-paying stock is entered into when the stock price is \$40 and the risk-free rate of interest is 10% per annum with continuous compounding.*

a. *What are the forward price and the initial value of the forward contract?*

b. *Six months later, the price of the stock is \$45 and the risk-free interest rate is still 10%. What are the forward price and the value of the forward contract?*

(a) The forward price,  $F_0$ , is given by equation (5.1) as:

$$F_0 = 40e^{0.1 \times 1} = 44.21$$

or \$44.21. The initial value of the forward contract is zero.

- (b) The delivery price  $K$  in the contract is \$44.21. The value of the contract,  $f$ , after six months is given by equation (5.5) as:

$$\begin{aligned} f &= 45 - 44.21e^{-0.1 \times 0.5} \\ &= 2.95 \end{aligned}$$

i.e., it is \$2.95. The forward price is:

$$45e^{0.1 \times 0.5} = 47.31$$

or \$47.31.

**Problem 5.10.**

*The risk-free rate of interest is 7% per annum with continuous compounding, and the dividend yield on a stock index is 3.2% per annum. The current value of the index is 150. What is the six-month futures price?*

Using equation (5.3) the six month futures price is

$$150e^{(0.07 - 0.032) \times 0.5} = 152.88$$

or \$152.88.

**Problem 5.11.**

*Assume that the risk-free interest rate is 9% per annum with continuous compounding and that the dividend yield on a stock index varies throughout the year. In February, May, August, and November, dividends are paid at a rate of 5% per annum. In other months, dividends are paid at a rate of 2% per annum. Suppose that the value of the index on July 31 is 1,300. What is the futures price for a contract deliverable on December 31 of the same year?*

The futures contract lasts for five months. The dividend yield is 2% for three of the months and 5% for two of the months. The average dividend yield is therefore

$$\frac{1}{5}(3 \times 2 + 2 \times 5) = 3.2\%$$

The futures price is therefore

$$1300e^{(0.09 - 0.032) \times 0.4167} = 1,331.80$$

or \$1331.80.

**Problem 5.12.**

Suppose that the risk-free interest rate is 10% per annum with continuous compounding and that the dividend yield on a stock index is 4% per annum. The index is standing at 400, and the futures price for a contract deliverable in four months is 405. What arbitrage opportunities does this create?

The theoretical futures price is

$$400e^{(0.10-0.04)\times 4/12} = 408.08$$

The actual futures price is only 405. This shows that the index futures price is too low relative to the index. The correct arbitrage strategy is

1. Buy futures contracts
2. Short the shares underlying the index.

**Problem 5.13.**

Estimate the difference between short-term interest rates in Mexico and the United States on January 8, 2007 from the information in Table 5.4.

The settlement prices for the futures contracts are

Jan	0.91250
Mar	0.91025

The March 2007 price is about 0.25% below the January 2007 price. This suggests that the short-term interest rate in the Mexico exceeded short-term interest rates in the United States by about 0.25% per two months or about 1.5% per year.

**Problem 5.14.**

The two-month interest rates in Switzerland and the United States are 2% and 5% per annum, respectively, with continuous compounding. The spot price of the Swiss franc is \$0.8000. The futures price for a contract deliverable in two months is \$0.8100. What arbitrage opportunities does this create?

The theoretical futures price is

$$0.8000e^{(0.05-0.02)\times 2/12} = 0.8040$$

The actual futures price is too high. This suggests that an arbitrageur should buy Swiss francs and short Swiss francs futures.

**Problem 5.15.**

The spot price of silver is \$9 per ounce. The storage costs are \$0.24 per ounce per year payable quarterly in advance. Assuming that interest rates are 10% per annum for all maturities, calculate the futures price of silver for delivery in nine months.

The present value of the storage costs for nine months are

$$0.06 + 0.06e^{-0.10\times 0.25} + 0.06e^{-0.10\times 0.5} = 0.176$$

or \$0.176. The futures price is from equation (5.11) given by  $F_0$  where

$$F_0 = (9.000 + 0.176)e^{0.1\times 0.75} = 9.89$$

i.e., it is \$9.89 per ounce.

**Problem 5.16.**

Suppose that  $F_1$  and  $F_2$  are two futures contracts on the same commodity with times to maturity,  $t_1$  and  $t_2$ , where  $t_2 > t_1$ . Prove that

$$F_2 \leq F_1 e^{r(t_2-t_1)}$$

where  $r$  is the interest rate (assumed constant) and there are no storage costs. For the purposes of this problem, assume that a futures contract is the same as a forward contract.

If

$$F_2 > F_1 e^{r(t_2-t_1)}$$

an investor could make a riskless profit by

1. Taking a long position in a futures contract which matures at time  $t_1$
2. Taking a short position in a futures contract which matures at time  $t_2$

When the first futures contract matures, the asset is purchased for  $F_1$  using funds borrowed at rate  $r$ . It is then held until time  $t_2$  at which point it is exchanged for  $F_2$  under the second contract. The costs of the funds borrowed and accumulated interest at time  $t_2$  is  $F_1 e^{r(t_2-t_1)}$ . A positive profit of

$$F_2 - F_1 e^{r(t_2-t_1)}$$

is then realized at time  $t_2$ . This type of arbitrage opportunity cannot exist for long. Hence:

$$F_2 \leq F_1 e^{r(t_2-t_1)}$$

**Problem 5.17.**

When a known future cash outflow in a foreign currency is hedged by a company using a forward contract, there is no foreign exchange risk. When it is hedged using futures contracts, the marking-to-market process does leave the company exposed to some risk. Explain the nature of this risk. In particular, consider whether the company is better off using a futures contract or a forward contract when

- a. The value of the foreign currency falls rapidly during the life of the contract
- b. The value of the foreign currency rises rapidly during the life of the contract
- c. The value of the foreign currency first rises and then falls back to its initial value
- d. The value of the foreign currency first falls and then rises back to its initial value

Assume that the forward price equals the futures price.

In total the gain or loss under a futures contract is equal to the gain or loss under the corresponding forward contract. However the timing of the cash flows is different. When the time value of money is taken into account a futures contract may prove to be more valuable or less valuable than a forward contract. Of course the company does not know in advance which will work out better. The long forward contract provides a perfect hedge. The long futures contract provides a slightly imperfect hedge.

- (a) In this case the forward contract would lead to a slightly better outcome. The company will make a loss on its hedge. If the hedge is with a forward contract the whole of

the loss will be realized at the end. If it is with a futures contract the loss will be realized day by day throughout the contract. On a present value basis the former is preferable.

- (b) In this case the futures contract would lead to a slightly better outcome. The company will make a gain on the hedge. If the hedge is with a forward contract the gain will be realized at the end. If it is with a futures contract the gain will be realized day by day throughout the life of the contract. On a present value basis the latter is preferable.
- (c) In this case the futures contract would lead to a slightly better outcome. This is because it would involve positive cash flows early and negative cash flows later.
- (d) In this case the forward contract would lead to a slightly better outcome. This is because, in the case of the futures contract, the early cash flows would be negative and the later cash flow would be positive.

**Problem 5.18.**

*It is sometimes argued that a forward exchange rate is an unbiased predictor of future exchange rates. Under what circumstances is this so?*

From the discussion in Section 5.14 of the text, the forward exchange rate is an unbiased predictor of the future exchange rate when the exchange rate has no systematic risk. To have no systematic risk the exchange rate must be uncorrelated with the return on the market.

**Problem 5.19.**

*Show that the growth rate in an index futures price equals the excess return of the index over the risk-free rate. Assume that the risk-free interest rate and the dividend yield are constant.*

Suppose that  $F_0$  is the futures price at time zero for a contract maturing at time  $T$  and  $F_1$  is the futures price for the same contract at time  $t_1$ . It follows that

$$F_0 = S_0 e^{(r-q)T}$$

$$F_1 = S_1 e^{(r-q)(T-t_1)}$$

where  $S_0$  and  $S_1$  are the spot price at times zero and  $t_1$ ,  $r$  is the risk-free rate, and  $q$  is the dividend yield. These equations imply that

$$\frac{F_1}{F_0} = \frac{S_1}{S_0} e^{-(r-q)t_1}$$

Define the excess return of the index over the risk-free rate as  $x$ . The total return is  $r + x$  and the return realized in the form of capital gains is  $r + x - q$ . It follows that  $S_1 = S_0 e^{(r+x-q)t_1}$  and the equation for  $F_1/F_0$  reduces to

$$\frac{F_1}{F_0} = e^{xt_1}$$

which is the required result.

**Problem 5.20.**

Show that equation (5.3) is true by considering an investment in the asset combined with a short position in a futures contract. Assume that all income from the asset is reinvested in the asset. Use an argument similar to that in footnotes 2 and 4 and explain in detail what an arbitrageur would do if equation (5.3) did not hold.

Suppose we buy  $N$  units of the asset and invest the income from the asset in the asset. The income from the asset causes our holding in the asset to grow at a continuously compounded rate  $q$ . By time  $T$  our holding has grown to  $Ne^{qT}$  units of the asset. Analogously to footnotes 2 and 4 of Chapter 5, we therefore buy  $N$  units of the asset at time zero at a cost of  $S_0$  per unit and enter into a forward contract to sell  $Ne^{qT}$  unit for  $F_0$  per unit at time  $T$ . This generates the following cash flows:

$$\begin{aligned} \text{Time 0: } & -NS_0 \\ \text{Time } T: & NF_0e^{qT} \end{aligned}$$

Because there is no uncertainty about these cash flows, the present value of the time  $T$  inflow must equal the time zero outflow when we discount at the risk-free rate. This means that

$$NS_0 = (NF_0e^{qT})e^{-rT}$$

or

$$F_0 = S_0e^{(r-q)T}$$

This is equation (5.3).

If  $F_0 > S_0e^{(r-q)T}$ , an arbitrageur should borrow money at rate  $r$  and buy  $N$  units of the asset. At the same time the arbitrageur should enter into a forward contract to sell  $Ne^{qT}$  units of the asset at time  $T$ . As income is received, it is reinvested in the asset. At time  $T$  the loan is repaid and the arbitrageur makes a profit of  $N(F_0e^{qT} - S_0e^{rT})$  at time  $T$ .

If  $F_0 < S_0e^{(r-q)T}$ , an arbitrageur should short  $N$  units of the asset investing the proceeds at rate  $r$ . At the same time the arbitrageur should enter into a forward contract to buy  $Ne^{qT}$  units of the asset at time  $T$ . When income is paid on the asset, the arbitrageur owes money on the short position. The investor meets this obligation from the cash proceeds of shorting further units. The result is that the number of units shorted grows at rate  $q$  to  $Ne^{qT}$ . The cumulative short position is closed out at time  $T$  and the arbitrageur makes a profit of  $N(S_0e^{rT} - F_0e^{qT})$ .

**Problem 5.21.**

Explain carefully what is meant by the expected price of a commodity on a particular future date. Suppose that the futures price of crude oil declines with the maturity of the contract at the rate of 2% per year. Assume that speculators tend to be short crude oil futures and hedgers tended to be long. What does the Keynes and Hicks argument imply about the expected future price of oil?

To understand the meaning of the expected future price of a commodity, suppose that there are  $N$  different possible prices at a particular future time:  $P_1, P_2, \dots, P_N$ . Define



$q_i$  as the (subjective) probability the price being  $P_i$  (with  $q_1 + q_2 + \dots + q_N = 1$ ). The expected future price is

$$\sum_{i=1}^N q_i P_i$$

Different people may have different expected future prices for the commodity. The expected future price in the market can be thought of as an average of the opinions of different market participants. Of course, in practice the actual price of the commodity at the future time may prove to be higher or lower than the expected price.

Keynes and Hicks argue that speculators on average make money from commodity futures trading and hedgers on average lose money from commodity futures trading. If speculators tend to have short positions in crude oil futures, the Keynes and Hicks argument implies that futures prices overstate expected future spot prices. If crude oil futures prices decline at 2% per year the Keynes and Hicks argument therefore implies an even faster decline for the expected price of crude oil.

**Problem 5.22.**

*The Value Line Index is designed to reflect changes in the value of a portfolio of over 1,600 equally weighted stocks. Prior to March 9, 1988, the change in the index from one day to the next was calculated as the geometric average of the changes in the prices of the stocks underlying the index. In these circumstances, does equation (5.8) correctly relate the futures price of the index to its cash price? If not, does the equation overstate or understate the futures price?*

When the geometric average of the price relatives is used, the changes in the value of the index do not correspond to changes in the value of a portfolio that is traded. Equation (5.8) is therefore no longer correct. The changes in the value of the portfolio is monitored by an index calculated from the arithmetic average of the prices of the stocks in the portfolio. Since the geometric average of a set of numbers is always less than the arithmetic average, equation (5.8) overstates the futures price. It is rumored that at one time (prior to 1988), equation (5.8) did hold for the Value Line Index. A major Wall Street firm was the first to recognize that this represented a trading opportunity. It made a financial killing by buying the stocks underlying the index and shorting the futures.

**Problem 5.23.**

*A U.S. company is interested in using the futures contracts traded on the CME to hedge its Australian dollar exposure. Define  $r$  as the interest rate (all maturities) on the U.S. dollar and  $r_f$  as the interest rate (all maturities) on the Australian dollar. Assume that  $r$  and  $r_f$  are constant and that the company uses a contract expiring at time  $T$  to hedge an exposure at time  $t$  ( $T > t$ ).*

a. *Show that the optimal hedge ratio is*

$$e^{(r_f - r)(T-t)}$$

b. *Show that, when  $t$  is one day, the optimal hedge ratio is almost exactly  $S_0/F_0$  where  $S_0$  is the current spot price of the currency and  $F_0$  is the current futures price of the currency for the contract maturing at time  $T$ .*

c. Show that the company can take account of the daily settlement of futures contracts for a hedge that lasts longer than one day by adjusting the hedge ratio so that it always equals the spot price of the currency divided by the futures price of the currency.

(a) The relationship between the futures price  $F_t$  and the spot price  $S_t$  at time  $t$  is

$$F_t = S_t e^{(r-r_f)(T-t)}$$

Suppose that the hedge ratio is  $h$ . The price obtained with hedging is

$$h(F_0 - F_t) + S_t$$

where  $F_0$  is the initial futures price. This is

$$hF_0 + S_t - hS_t e^{(r-r_f)(T-t)}$$

If  $h = e^{(r_f-r)(T-t)}$ , this reduces to  $hF_0$  and a zero variance hedge is obtained.

(b) When  $t$  is one day,  $h$  is approximately  $e^{(r_f-r)T} = S_0/F_0$ . The appropriate hedge ratio is therefore  $S_0/F_0$ .

(c) When a futures contract is used for hedging, the price movements in each day should in theory be hedged separately. This is because the daily settlement means that a futures contract is closed out and rewritten at the end of each day. From (b) the correct hedge ratio at any given time is, therefore,  $S/F$  where  $S$  is the spot price and  $F$  is the futures price. Suppose there is an exposure to  $N$  units of the foreign currency and  $M$  units of the foreign currency underlie one futures contract. With a hedge ratio of 1 we should trade  $N/M$  contracts. With a hedge ratio of  $S/F$  we should trade

$$\frac{SN}{FM}$$

contracts. In other words we should calculate the number of contracts that should be traded as the dollar value of our exposure divided by the dollar value of one futures contract (This is not the same as the dollar value of our exposure divided by the dollar value of the assets underlying one futures contract.) Since a futures contract is settled daily, we should in theory rebalance our hedge daily so that the outstanding number of futures contracts is always  $(SN)/(FM)$ . This is known as tailing the hedge. (See Section 3.4 of the text.)

## ASSIGNMENT QUESTIONS

### Problem 5.24.

A stock is expected to pay a dividend of \$1 per share in two months and in five months. The stock price is \$50, and the risk-free rate of interest is 8% per annum with

continuous compounding for all maturities. An investor has just taken a short position in a six-month forward contract on the stock.

a. What are the forward price and the initial value of the forward contract?

b. Three months later, the price of the stock is \$48 and the risk-free rate of interest is still 8% per annum. What are the forward price and the value of the short position in the forward contract?

(a) The present value,  $I$ , of the income from the security is given by:

$$I = 1 \times e^{-0.08 \times 2/12} + 1 \times e^{-0.08 \times 5/12} = 1.9540$$

From equation (5.2) the forward price,  $F_0$ , is given by:

$$F_0 = (50 - 1.9540)e^{0.08 \times 0.5} = 50.01$$

or \$50.01. The initial value of the forward contract is (by design) zero. The fact that the forward price is very close to the spot price should come as no surprise. When the compounding frequency is ignored the dividend yield on the stock equals the risk-free rate of interest.

(b) In three months:

$$I = e^{-0.08 \times 2/12} = 0.9868$$

The delivery price,  $K$ , is 50.01. From equation (5.6) the value of the short forward contract,  $f$ , is given by

$$f = -(48 - 0.9868 - 50.01e^{-0.08 \times 3/12}) = 2.01$$

and the forward price is

$$(48 - 0.9868)e^{0.08 \times 3/12} = 47.96$$

### Problem 5.25.

A bank offers a corporate client a choice between borrowing cash at 11% per annum and borrowing gold at 2% per annum. (If gold is borrowed, interest must be repaid in gold. Thus, 100 ounces borrowed today would require 102 ounces to be repaid in one year.) The risk-free interest rate is 9.25% per annum, and storage costs are 0.5% per annum. Discuss whether the rate of interest on the gold loan is too high or too low in relation to the rate of interest on the cash loan. The interest rates on the two loans are expressed with annual compounding. The risk-free interest rate and storage costs are expressed with continuous compounding.

My explanation of this problem to students usually goes as follows. Suppose that the price of gold is \$550 per ounce and the corporate client wants to borrow \$550,000. The client has a choice between borrowing \$550,000 in the usual way and borrowing 1,000 ounces of gold. If it borrows \$550,000 in the usual way, an amount equal to  $550,000 \times 1.11 =$

\$610,500 must be repaid. If it borrows 1,000 ounces of gold it must repay 1,020 ounces. In equation (5.12),  $r = 0.0925$  and  $u = 0.005$  so that the forward price is

$$550e^{(0.0925+0.005)\times 1} = 606.33$$

By buying 1,020 ounces of gold in the forward market the corporate client can ensure that the repayment of the gold loan costs

$$1,020 \times 606.33 = \$618,457$$

Clearly the cash loan is the better deal ( $618,457 > 610,500$ ).

This argument shows that the rate of interest on the gold loan is too high. What is the correct rate of interest? Suppose that  $R$  is the rate of interest on the gold loan. The client must repay  $1,000(1 + R)$  ounces of gold. When forward contracts are used the cost of this is

$$1,000(1 + R) \times 606.33$$

This equals the \$610,500 required on the cash loan when  $R = 0.688\%$ . The rate of interest on the gold loan is too high by about 1.31%. However, this might be simply a reflection of the higher administrative costs incurred with a gold loan.

It is interesting to note that this is not an artificial question. Many banks are prepared to make gold loans at interest rates of about 2% per annum.

#### **Problem 5.26.**

*A company that is uncertain about the exact date when it will pay or receive a foreign currency may try to negotiate with its bank a forward contract that specifies a period during which delivery can be made. The company wants to reserve the right to choose the exact delivery date to fit in with its own cash flows. Put yourself in the position of the bank. How would you price the product that the company wants?*

It is likely that the bank will price the product on assumption that the company chooses the delivery date least favorable to the bank. If the foreign interest rate is higher than the domestic interest rate then

1. The earliest delivery date will be assumed when the company has a long position.
2. The latest delivery date will be assumed when the company has a short position.

If the foreign interest rate is lower than the domestic interest rate then

1. The latest delivery date will be assumed when the company has a long position.
2. The earliest delivery date will be assumed when the company has a short position.

If the company chooses a delivery which, from a purely financial viewpoint, is suboptimal the bank makes a gain.

#### **Problem 5.27.**

*A trader owns gold as part of a long-term investment portfolio. The trader can buy gold for \$550 per ounce and sell gold for \$549 per ounce. The trader can borrow funds at 6% per year and invest funds at 5.5% per year. (Both interest rates are expressed with*

annual compounding.) For what range of one-year forward prices of gold does the trader have no arbitrage opportunities? Assume there is no bid-offer spread for forward prices.

Suppose that  $F_0$  is the one-year forward price of gold. If  $F_0$  is relatively high, the trader can borrow \$550 at 6%, buy one ounce of gold and enter into a forward contract to sell gold in one year for  $F_0$ . The profit made in one year is

$$F_0 - 550 \times 1.06 = F_0 - 583$$

If  $F_0$  is relatively low, the trader can sell one ounce of gold for \$549, invest the proceeds at 5.5%, and enter into a forward contract to buy the gold back for  $F_0$ . The profit (relative to the position the trader would be in if the gold were held in the portfolio during the year) is

$$549 \times 1.055 - F_0 = 579.195 - F_0$$

This shows that there is no arbitrage opportunity if the forward price is between \$579.195 and \$583 per ounce.

**Problem 5.28.**

A company enters into a forward contract with a bank to sell a foreign currency for  $K_1$  at time  $T_1$ . The exchange rate at time  $T_1$  proves to be  $S_1$  ( $> K_1$ ). The company asks the bank if it can roll the contract forward until time  $T_2$  ( $> T_1$ ) rather than settle at time  $T_1$ . The bank agrees to a new delivery price,  $K_2$ . Explain how  $K_2$  should be calculated.

The value of the contract to the bank at time  $T_1$  is  $S_1 - K_1$ . The bank will choose  $K_2$  so that the new (rolled forward) contract has a value of  $S_1 - K_1$ . This means that

$$S_1 e^{-r_f(T_2-T_1)} - K_2 e^{-r(T_2-T_1)} = S_1 - K_1$$

where  $r$  and  $r_f$  are the domestic and foreign risk-free rate observed at time  $T_1$  and applicable to the period between time  $T_1$  and  $T_2$ . This means that

$$K_2 = S_1 e^{(r-r_f)(T_2-T_1)} - (S_1 - K_1) e^{r(T_2-T_1)}$$

This equation shows that there are two components to  $K_2$ . The first is the forward price at time  $T_1$ . The second is an adjustment to the forward price equal to the bank's gain on the first part of the contract compounded forward at the domestic risk-free rate.

# CHAPTER 6

## Interest Rate Futures

### Notes for the Instructor

This chapter discusses how interest rate futures contracts are quoted, how they work, and how they are used for hedging. I start by discussing the material in Sections 6.1 and 6.2 on day counts and how prices are quoted in the spot market. (It is fun to talk about Business Snapshot 6.1 when day count conventions are discussed.) I like to spend some time making sure students are comfortable with the Treasury bond futures contracts and the Eurodollar futures contract. In the case of the Treasury bond futures contract they should understand where conversion factors come from, the cheapest-to-deliver bond calculations, and the wild card play (see Business Snapshot 6.2). In the case of Eurodollar futures they should understand the quotation system, that the contract's value changes by \$25 for each basis point change in the quote, and how the final cash settlement works. In the slides I have included a numerical example help explain this.

Students should also appreciate that a convexity adjustment is necessary to calculate a forward rate from a Eurodollar futures quote. They will not at this stage understand where equation 6.3 comes from, but they should understand that there are two reasons why forward and futures interest rates are different. The first is that futures are settled daily; forwards are not. The second is that futures (if not daily settled) would provide a payoff at the beginning of the period covered by the rate; forwards provide a payoff at the end of the period covered by the rate.

The final part of the chapter covers the use of interest rate futures for duration-based hedging. I usually illustrate this material with a numerical example.

I sometimes use Problem 6.23 in class to help explain how Eurodollar futures contracts work and the impact of day count conventions. (Without adjusting for the day count convention, the arbitrage opportunity appears to be the other way round.) Problems 6.24 and 6.26 can be used as assignment questions.

### QUESTIONS AND PROBLEMS

#### Problem 6.1.

*A U.S. Treasury bond pays a 7% coupon on January 7 and July 7. How much interest accrue per \$100 of principal to the bond holder between July 7, 2009 and August 9, 2009? How would your answer be different if it were a corporate bond?*

There are 33 calendar days between July 7, 2009 and August 9, 2009. There are 184 calendar days between July 7, 2009 and January 7, 2010. The interest earned per \$100 of principal is therefore  $3.5 \times 33/184 = \$0.6277$ . For a corporate bond we assume 32 days

between July 7 and August 9, 2009 and 180 days between July 7, 2009 and January 7, 2010. The interest earned is  $3.5 \times 32/180 = \$0.6222$ .

**Problem 6.2.**

*It is January 9, 2009. The price of a Treasury bond with a 12% coupon that matures on October 12, 2020, is quoted as 102-07. What is the cash price?*

There are 89 days between October 12, 2009, and January 9, 2010. There are 182 days between October 12, 2009, and April 12, 2010. The cash price of the bond is obtained by adding the accrued interest to the quoted price. The quoted price is  $102\frac{7}{32}$  or 102.21875. The cash price is therefore

$$102.21875 + \frac{89}{182} \times 6 = \$105.15$$

**Problem 6.3.**

*How is the conversion factor of a bond calculated by the Chicago Board of Trade? How is it used?*

The conversion factor for a bond is equal to the quoted price the bond would have per dollar of principal on the first day of the delivery month on the assumption that the interest rate for all maturities equals 6% per annum (with semiannual compounding). The bond maturity and the times to the coupon payment dates are rounded down to the nearest three months for the purposes of the calculation. The conversion factor defines how much an investor with a short bond futures contract receives when bonds are delivered. If the conversion factor is 1.2345 the amount investor receives is calculated by multiplying 1.2345 by the most recent futures price and adding accrued interest.

**Problem 6.4.**

*A Eurodollar futures price changes from 96.76 to 96.82. What is the gain or loss to an investor who is long two contracts?*

The Eurodollar futures price has increased by 6 basis points. The investor makes a gain per contract of  $25 \times 6 = \$150$  or \$300 in total.

**Problem 6.5.**

*What is the purpose of the convexity adjustment made to Eurodollar futures rates? Why is the convexity adjustment necessary?*

Suppose that a Eurodollar futures quote is 95.00. This gives a futures rate of 5% for the three-month period covered by the contract. The convexity adjustment is the amount by which futures rate has to be reduced to give an estimate of the forward rate for the period. The convexity adjustment is necessary because a) the futures contract is settled daily and b) the futures contract expires at the beginning of the three months. Both of these lead to the futures rate being greater than the forward rate.

**Problem 6.6.**

The 350-day LIBOR rate is 3% with continuous compounding and the forward rate calculated from a Eurodollar futures contract that matures in 350 days is 3.2% with continuous compounding. Estimate the 440-day zero rate.

From equation (6.4) the rate is

$$\frac{3.2 \times 90 + 3 \times 350}{440} = 3.0409$$

or 3.0409%.

**Problem 6.7.**

It is January 30. You are managing a bond portfolio worth \$6 million. The duration of the portfolio in six months will be 8.2 years. The September Treasury bond futures price is currently 108-15, and the cheapest-to-deliver bond will have a duration of 7.6 years in September. How should you hedge against changes in interest rates over the next six months?

The value of a contract is  $108\frac{15}{32} \times 1,000 = \$108,468.75$ . The number of contracts that should be shorted is

$$\frac{6,000,000}{108,468.75} \times \frac{8.2}{7.6} = 59.7$$

Rounding to the nearest whole number, 60 contracts should be shorted. The position should be closed out at the end of July.

**Problem 6.8.**

The price of a 90-day Treasury bill is quoted as 10.00. What continuously compounded return (on an actual/365 basis) does an investor earn on the Treasury bill for the 90-day period?

The cash price of the Treasury bill is

$$100 - \frac{90}{360} \times 10 = \$97.50$$

The annualized continuously compounded return is

$$\frac{365}{90} \ln \left( 1 + \frac{2.5}{97.5} \right) = 10.27\%$$

**Problem 6.9.**

It is May 5, 2008. The quoted price of a government bond with a 12% coupon that matures on July 27, 2011, is 110-17. What is the cash price?

The number of days between January 27, 2008 and May 5, 2008 is 99. The number of days between January 27, 2008 and July 27, 2008 is 182. The accrued interest is therefore

$$6 \times \frac{99}{182} = 3.2637$$



The quoted price is 110.5312. The cash price is therefore

$$110.5312 + 3.2637 = 113.7949$$

or \$113.79.

**Problem 6.10.**

Suppose that the Treasury bond futures price is 101-12. Which of the following four bonds is cheapest to deliver?

Bond	Price	Conversion Factor
1	125-05	1.2131
2	142-15	1.3792
3	115-31	1.1149
4	144-02	1.4026

The cheapest-to-deliver bond is the one for which

$$\text{Quoted Price} - \text{Futures Price} \times \text{Conversion Factor}$$

is least. Calculating this factor for each of the 4 bonds we get

$$\text{Bond 1: } 125.15625 - 101.375 \times 1.2131 = 2.178$$

$$\text{Bond 2: } 142.46875 - 101.375 \times 1.3792 = 2.652$$

$$\text{Bond 3: } 115.96875 - 101.375 \times 1.1149 = 2.946$$

$$\text{Bond 4: } 144.06250 - 101.375 \times 1.4026 = 1.874$$

Bond 4 is therefore the cheapest to deliver.

**Problem 6.11.**

It is July 30, 2009. The cheapest-to-deliver bond in a September 2009 Treasury bond futures contract is a 13% coupon bond, and delivery is expected to be made on September 30, 2009. Coupon payments on the bond are made on February 4 and August 4 each year. The term structure is flat, and the rate of interest with semiannual compounding is 12% per annum. The conversion factor for the bond is 1.5. The current quoted bond price is \$110. Calculate the quoted futures price for the contract.

There are 176 days between February 4 and July 30 and 181 days between February 4 and August 4. The cash price of the bond is, therefore:

$$110 + \frac{176}{181} \times 6.5 = 116.32$$

The rate of interest with continuous compounding is  $2 \ln 1.06 = 0.1165$  or 11.65% per annum. A coupon of 6.5 will be received in 5 days (= 0.01370 years) time. The present value of the coupon is

$$6.5e^{-0.01370 \times 0.1165} = 6.490$$

The futures contract lasts for 62 days (= 0.1699 years). The cash futures price if the contract were written on the 13% bond would be

$$(116.32 - 6.490)e^{0.1699 \times 0.1165} = 112.03$$

At delivery there are 57 days of accrued interest. The quoted futures price if the contract were written on the 13% bond would therefore be

$$112.03 - 6.5 \times \frac{57}{184} = 110.01$$

Taking the conversion factor into account the quoted futures price should be:

$$\frac{110.01}{1.5} = 73.34$$

**Problem 6.12.**

*An investor is looking for arbitrage opportunities in the Treasury bond futures market. What complications are created by the fact that the party with a short position can choose to deliver any bond with a maturity of over 15 years?*

If the bond to be delivered and the time of delivery were known, arbitrage would be straightforward. When the futures price is too high, the arbitrageur buys bonds and shorts an equivalent number of bond futures contracts. When the futures price is too low, the arbitrageur sells bonds and goes long an equivalent number of bond futures contracts.

Uncertainty as to which bond will be delivered introduces complications. The bond that appears cheapest-to-deliver now may not in fact be cheapest-to-deliver at maturity. In the case where the futures price is too high, this is not a major problem since the party with the short position (i.e., the arbitrageur) determines which bond is to be delivered. In the case where the futures price is too low, the arbitrageur's position is far more difficult since he or she does not know which bond to buy; it is unlikely that a profit can be locked in for all possible outcomes.

**Problem 6.13.**

*Suppose that the nine-month LIBOR interest rate is 8% per annum and the six-month LIBOR interest rate is 7.5% per annum (both with actual/365 and continuous compounding). Estimate the three-month Eurodollar futures price quote for a contract maturing in six months.*

The forward interest rate for the time period between months 6 and 9 is 9% per annum with continuous compounding. This is because 9% per annum for three months

when combined with  $7\frac{1}{2}\%$  per annum for six months gives an average interest rate of 8% per annum for the nine-month period.

With quarterly compounding the forward interest rate is

$$4(e^{0.09/4} - 1) = 0.09102$$

or 9.102%. This assumes that the day count is actual/actual. With a day count of actual/360 the rate is  $9.102 \times 360/365 = 8.977$ . The three-month Eurodollar quote for a contract maturing in six months is therefore

$$100 - 8.977 = 91.02$$

This assumes no difference between futures and forward prices.

#### **Problem 6.14.**

*Suppose that the 300-day LIBOR zero rate is 4% and Eurodollar quotes for contracts maturing in 300, 398 and 489 days are 95.83, 95.62, and 95.48. Calculate 398-day and 489-day LIBOR zero rates. Assume no difference between forward and futures rates for the purposes of your calculations.*

The forward rates calculated from the first two Eurodollar futures are 4.17% and 4.38%. These are expressed with an actual/360 day count and quarterly compounding. With continuous compounding and an actual/365 day count they are  $(365/90)\ln(1 + 0.0417/4) = 4.2060\%$  and  $(365/90)\ln(1 + 0.0438/4) = 4.4167\%$ . It follows from equation (6.4) that the 398 day rate is

$$\frac{4 \times 300 + 4.2060 \times 98}{398} = 4.0507$$

or 4.0507%. The 489 day rate is

$$\frac{4.0507 \times 398 + 4.4167 \times 91}{489} = 4.1188$$

or 4.1188%. We are assuming that the first futures rate applies to 98 days rather than the usual 91 days. The third futures quote is not needed.

#### **Problem 6.15.**

*Suppose that a bond portfolio with a duration of 12 years is hedged using a futures contract in which the underlying asset has a duration of four years. What is likely to be the impact on the hedge of the fact that the 12-year rate is less volatile than the four-year rate?*

Duration-based hedging schemes assume parallel shifts in the yield curve. Since the 12-year rate tends to move by less than the 4-year rate, the portfolio manager may find that he or she is over-hedged.

**Problem 6.16.**

Suppose that it is February 20 and a treasurer realizes that on July 17 the company will have to issue \$5 million of commercial paper with a maturity of 180 days. If the paper were issued today, the company would realize \$4,820,000. (In other words, the company would receive \$4,820,000 for its paper and have to redeem it at \$5,000,000 in 180 days' time.) The September Eurodollar futures price is quoted as 92.00. How should the treasurer hedge the company's exposure?

The company treasurer can hedge the company's exposure by shorting Eurodollar futures contracts. The Eurodollar futures position leads to a profit if rates rise and a loss if they fall.

The duration of the commercial paper is twice that of the Eurodollar deposit underlying the Eurodollar futures contract. The contract price of a Eurodollar futures contract is 980,000. The number of contracts that should be shorted is, therefore,

$$\frac{4,820,000}{980,000} \times 2 = 9.84$$

Rounding to the nearest whole number 10 contracts should be shorted.

**Problem 6.17.**

On August 1 a portfolio manager has a bond portfolio worth \$10 million. The duration of the portfolio in October will be 7.1 years. The December Treasury bond futures price is currently 91-12 and the cheapest-to-deliver bond will have a duration of 8.8 years at maturity. How should the portfolio manager immunize the portfolio against changes in interest rates over the next two months?

The treasurer should short Treasury bond futures contract. If bond prices go down, this futures position will provide offsetting gains. The number of contracts that should be shorted is

$$\frac{10,000,000 \times 7.1}{91,375 \times 8.8} = 88.30$$

Rounding to the nearest whole number 88 contracts should be shorted.

**Problem 6.18.**

How can the portfolio manager change the duration of the portfolio to 3.0 years in Problem 6.17?

The answer in Problem 6.17 is designed to reduce the duration to zero. To reduce the duration from 7.1 to 3.0 instead of from 7.1 to 0, the treasurer should short

$$\frac{4.1}{7.1} \times 88.30 = 50.99$$

or 51 contracts.

**Problem 6.19.**

Between October 30, 2009, and November 1, 2009, you have a choice between owning a U.S. government bond paying a 12% coupon and a U.S. corporate bond paying a 12% coupon. Consider carefully the day count conventions discussed in this chapter and decide which of the two bonds you would prefer to own. Ignore the risk of default.

You would prefer to own the Treasury bond. Under the 30/360 day count convention there is one day between October 30, 2009 and November 1, 2009. Under the actual/actual (in period) day count convention, there are two days. Therefore you would earn approximately twice as much interest by holding the Treasury bond. This assumes that the quoted prices of the two bonds are the same.

**Problem 6.20.**

Suppose that a Eurodollar futures quote is 88 for a contract maturing in 60 days. What is the LIBOR forward rate for the 60- to 150-day period? Ignore the difference between futures and forwards for the purposes of this question.

The Eurodollar futures contract price of 88 means that the Eurodollar futures rate is 12% per annum. This is the forward rate for the 60- to 150-day period with quarterly compounding and an actual/360 day count convention.

**Problem 6.21.**

The three-month Eurodollar futures price for a contract maturing in six years is quoted as 95.20. The standard deviation of the change in the short-term interest rate in one year is 1.1%. Estimate the forward LIBOR interest rate for the period between 6.00 and 6.25 years in the future.

Using the notation of Section 6.4,  $\sigma = 0.011$ ,  $T_1 = 6$ , and  $T_2 = 6.25$ . The convexity adjustment is

$$\frac{1}{2} \times 0.011^2 \times 6 \times 6.25 = 0.002269$$

or about 23 basis points. The futures rate is 4.8% with quarterly compounding and an actual/360 day count.  $(365/90)\ln(1.012) = 0.0484$  or 4.84% with continuous compounding and actual/365 day count. The forward rate is therefore  $4.84 - 0.23 = 4.61\%$  with continuous compounding.

**Problem 6.22.**

Explain why the forward interest rate is less than the corresponding futures interest rate calculated from a Eurodollar futures contract.

Suppose that the contracts apply to the interest rate between times  $T_1$  and  $T_2$ . There are two reasons for a difference between the forward rate and the futures rate. The first is that the futures contract is settled daily whereas the forward contract is settled once at time  $T_2$ . The second is that without daily settlement a futures contract would be settled at time  $T_1$  not  $T_2$ . Both reasons tend to make the futures rate greater than the forward rate.

## ASSIGNMENT QUESTIONS

### Problem 6.23.

Assume that a bank can borrow or lend money at the same interest rate in the LIBOR market. The 90-day rate is 10% per annum, and the 180-day rate is 10.2% per annum, both expressed with continuous compounding and actual/actual day count. The Eurodollar futures price for a contract maturing in 91 days is quoted as 89.5. What arbitrage opportunities are open to the bank?

The Eurodollar futures contract price of 89.5 means that the Eurodollar futures rate is 10.5% per annum with quarterly compounding and an actual/360 day count. This becomes  $10.5 \times 365/360 = 10.646\%$  with an actual/actual day count. This is

$$4 \ln(1 + 0.25 \times 0.10646) = 0.1051$$

or 10.51% with continuous compounding. The forward rate given by the 91-day rate and the 182-day rate is 10.4% with continuous compounding. This suggests the following arbitrage opportunity:

1. Buy Eurodollar futures.
2. Borrow 182-day money.
3. Invest the borrowed money for 91 days.

### Problem 6.24.

A Canadian company wishes to create a Canadian LIBOR futures contract from a U.S. Eurodollar futures contract and forward contracts on foreign exchange. Using an example, explain how the company should proceed. For the purposes of this problem, assume that a futures contract is the same as a forward contract.

The U.S. Eurodollar futures contract maturing at time  $T$  enables an investor to lock in the forward rate for the period between  $T$  and  $T^*$  where  $T^*$  is three months later than  $T$ . If  $\hat{r}$  is the forward rate, the U.S. dollar cash flows that can be locked in are

$$\begin{array}{ll} -Ae^{-\hat{r}(T^*-T)} & \text{at time } T \\ +A & \text{at time } T^* \end{array}$$

where  $A$  is the principal amount. To convert these to Canadian dollar cash flows, the Canadian company must enter into a short forward foreign exchange contract to sell Canadian dollars at time  $T$  and a long forward foreign exchange contract to buy Canadian dollars at time  $T^*$ . Suppose  $F$  and  $F^*$  are the forward exchange rates for contracts maturing at times  $T$  and  $T^*$ . (These represent the number of Canadian dollars per U.S. dollar.) The Canadian dollars to be sold at time  $T$  are

$$Ae^{-\hat{r}(T^*-T)} F$$

and the Canadian dollars to be purchased at time  $T^*$  are

$$AF^*$$

The forward contracts convert the U.S. dollar cash flows to the following Canadian dollar cash flows:

$$\begin{array}{ll} -Ae^{-\hat{r}(T^*-T)}F & \text{at time } T \\ +AF^* & \text{at time } T^* \end{array}$$

This is a Canadian dollar LIBOR futures contract where the principal amount is  $AF^*$ .

**Problem 6.25.**

The futures price for the June 2009 CBOT bond futures contract is 118-23.

- a. Calculate the conversion factor for a bond maturing on January 1, 2025, paying a coupon of 10%.
  - b. Calculate the conversion factor for a bond maturing on October 1, 2030, paying a coupon of 7%.
  - c. Suppose that the quoted prices of the bonds in (a) and (b) are 169.00 and 136.00, respectively. Which bond is cheaper to deliver?
  - d. Assuming that the cheapest-to-deliver bond is actually delivered, what is the cash price received for the bond?
- (a) On the first day of the delivery month the bond has 15 years and 7 months to maturity. The value of the bond assuming it lasts 15.5 years and all rates are 6% per annum with semiannual compounding is

$$\sum_{i=1}^{31} \frac{5}{1.03^i} + \frac{100}{1.03^{31}} = 140.00$$

The conversion factor is therefore 1.4000.

- (b) On the first day of the delivery month the bond has 21 years and 4 months to maturity. The value of the bond assuming it lasts 21.25 years and all rates are 6% per annum with semiannual compounding is

$$\frac{1}{\sqrt{1.03}} \left[ 3.5 + \sum_{i=1}^{42} \frac{3.5}{1.03^i} + \frac{100}{1.03^{42}} \right] = 113.66$$

Subtracting the accrued interest of 1.75, this becomes 111.91. The conversion factor is therefore 1.1191.

- (c) For the first bond, the quoted futures price times the conversion factor is

$$118.71825 \times 1.4000 = 166.2056$$

This is 2.7944 less than the quoted bond price. For the second bond, the quoted futures price times the conversion factor is

$$118.71825 \times 1.1191 = 132.8576$$

This is 3.1424 less than the quoted bond price. The first bond is therefore the cheapest to deliver.

- (d) The price received for the bond is 166.2056 plus accrued interest.<sup>1</sup> There are 176 days between January 1, 2009 and June 25, 2009. There are 181 days between January 1, 2009 and July 1, 2009. The accrued interest is therefore

$$5 \times \frac{176}{181} = 4.8619$$

The cash price received for the bond is therefore 171.0675.

**Problem 6.26.**

*A portfolio manager plans to use a Treasury bond futures contract to hedge a bond portfolio over the next three months. The portfolio is worth \$100 million and will have a duration of 4.0 years in three months. The futures price is 122, and each futures contract is on \$100,000 of bonds. The bond that is expected to be cheapest to deliver will have a duration of 9.0 years at the maturity of the futures contract. What position in futures contracts is required?*

- a. *What adjustments to the hedge are necessary if after one month the bond that is expected to be cheapest to deliver changes to one with a duration of seven years?*
- b. *Suppose that all rates increase over the next three months, but long-term rates increase less than short-term and medium-term rates. What is the effect of this on the performance of the hedge?*

The number of short futures contracts required is

$$\frac{100,000,000 \times 4.0}{122,000 \times 9.0} = 364.3$$

Rounding to the nearest whole number 364 contracts should be shorted.

- (a) This increases the number of contracts that should be shorted to

$$\frac{100,000,000 \times 4.0}{122,000 \times 7.0} = 468.4$$

or 468 when we round to the nearest whole number.

- (b) In this case the gain on the short futures position is likely to be less than the loss on the bond portfolio. This is because the gain on the short futures position depends on the size of the movement in long-term rates and the loss on the bond portfolio depends on the size of the movement in medium-term rates. Duration-based hedging assumes that the movements in the two rates are the same.

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<sup>1</sup> Note that the delivery date was not specified in the first printing of the book. We assume it is June 25.



# CHAPTER 7

## Swaps

### Notes for the Instructor

This chapter covers the nature of swaps and how they are valued. I believe that it makes sense to teach swaps soon after forward contracts are covered because a swap is nothing more than a convenient way of bundling forward contracts. The growth of the swaps since the early 1980s makes them one of the most important derivative instruments. This chapter covers interest rate and currency swaps and provides a brief review of nonstandard swaps. More details on nonstandard swaps are in Chapter 32.

After explaining how swaps work and the way they can be used to transform assets and liabilities, I present the traditional comparative advantage argument for plain vanilla interest rate swaps and then proceed to explain why it is flawed. This usually generates a lively discussion. The key point is that the comparative advantage argument compares apples with oranges. Suppose a BBB-rated company wants to borrow at a fixed rate for five years and can choose between a fixed rate of 8% and a floating rate of LIBOR+1%. Borrowing floating and swapping to fixed appears attractive. But this ignores a key point. A fixed-rate loan will lead to exactly the same rate of interest applying each year for five years. By contrast, the spread over LIBOR on the floating-rate loan is usually guaranteed for only 6 months. If the creditworthiness of the company declines, the rate is liable to increase when the loan is rolled over. This means that borrowing floating and swapping to fixed subjects the BBB to “rollover risk”. If a financial institution offers LIBOR+1% and guarantees that the spread over LIBOR will not change, we are comparing apples with apples. However, when a table similar to 7.4 is constructed, there is then found to be no comparative advantage.

A useful exercise is to take a situation such as that shown in Figure 7.7 and ask students to identify the credit risk and rollover risk of AAACorp, BBBCorp, and the financial institution.

This is the time when the nature of the LIBOR/swap zero curve can be explained to students. I go over the arguments in Section 7.5 carefully and explain the procedure (outlined in Section 7.6) for calculating the LIBOR/swap zero curve.

In the case of currency swaps the exchange of principal needs to be explained. Valuation methods are structurally very similar to those for interest rate swaps and can usually be covered fairly quickly.

Problems 7.19, 7.20, 7.21, 7.22, and 7.23 all work well as assignment questions. I usually ask students to hand in two of them.

## QUESTIONS AND PROBLEMS

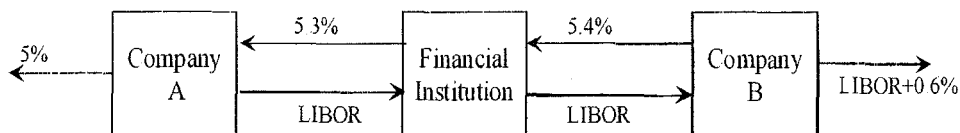
### Problem 7.1.

Companies A and B have been offered the following rates per annum on a \$20 million five-year loan:

	Fixed Rate	Floating Rate
Company A	5.0%	LIBOR + 0.1%
Company B	6.4%	LIBOR + 0.6%

Company A requires a floating-rate loan; company B requires a fixed-rate loan. Design a swap that will net a bank, acting as intermediary, 0.1% per annum and that will appear equally attractive to both companies.

A has an apparent comparative advantage in fixed-rate markets but wants to borrow floating. B has an apparent comparative advantage in floating-rate markets but wants to borrow fixed. This provides the basis for the swap. There is a 1.4% per annum differential between the fixed rates offered to the two companies and a 0.5% per annum differential between the floating rates offered to the two companies. The total gain to all parties from the swap is therefore  $1.4 - 0.5 = 0.9\%$  per annum. Because the bank gets 0.1% per annum of this gain, the swap should make each of A and B 0.4% per annum better off. This means that it should lead to A borrowing at  $\text{LIBOR} - 0.3\%$  and to B borrowing at 6.0%. The appropriate arrangement is therefore as shown in Figure S7.1.



**Figure S7.1** Swap for Problem 7.1

### Problem 7.2.

Company X wishes to borrow U.S. dollars at a fixed rate of interest. Company Y wishes to borrow Japanese yen at a fixed rate of interest. The amounts required by the two companies are roughly the same at the current exchange rate. The companies have been quoted the following interest rates, which have been adjusted for the impact of taxes:

	Yen	Dollars
Company X	5.0%	9.6%
Company Y	6.5%	10.0%

Design a swap that will net a bank, acting as intermediary, 50 basis points per annum. Make the swap equally attractive to the two companies and ensure that all foreign exchange risk is assumed by the bank.

X has a comparative advantage in yen markets but wants to borrow dollars. Y has a comparative advantage in dollar markets but wants to borrow yen. This provides the basis for the swap. There is a 1.5% per annum differential between the yen rates and a 0.4% per annum differential between the dollar rates. The total gain to all parties from the swap is therefore  $1.5 - 0.4 = 1.1\%$  per annum. The bank requires 0.5% per annum, leaving 0.3% per annum for each of X and Y. The swap should lead to X borrowing dollars at  $9.6 - 0.3 = 9.3\%$  per annum and to Y borrowing yen at  $6.5 - 0.3 = 6.2\%$  per annum. The appropriate arrangement is therefore as shown in Figure S7.2. All foreign exchange risk is borne by the bank.

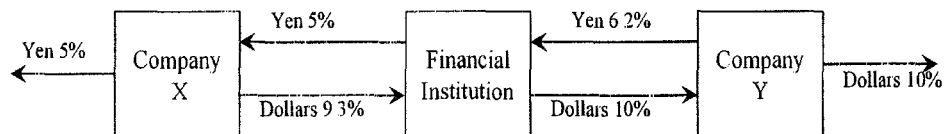


Figure S7.2 Swap for Problem 7.2

### Problem 7.3.

A \$100 million interest rate swap has a remaining life of 10 months. Under the terms of the swap, six-month LIBOR is exchanged for 7% per annum (compounded semiannually). The average of the bid-offer rate being exchanged for six-month LIBOR in swaps of all maturities is currently 5% per annum with continuous compounding. The six-month LIBOR rate was 4.6% per annum two months ago. What is the current value of the swap to the party paying floating? What is its value to the party paying fixed?

In four months \$3.5 million ( $= 0.5 \times 0.07 \times \$100$  million) will be received and \$2.3 million ( $= 0.5 \times 0.046 \times \$100$  million) will be paid. (We ignore day count issues.) In 10 months \$3.5 million will be received, and the LIBOR rate prevailing in four months' time will be paid. The value of the fixed-rate bond underlying the swap is

$$3.5e^{-0.05 \times 4/12} + 103.5e^{-0.05 \times 10/12} = \$102.718 \text{ million}$$

The value of the floating-rate bond underlying the swap is

$$(100 + 2.3)e^{-0.05 \times 4/12} = \$100.609 \text{ million}$$

The value of the swap to the party paying floating is  $\$102.718 - \$100.609 = \$2.109$  million. The value of the swap to the party paying fixed is  $-\$2.109$  million.

These results can also be derived by decomposing the swap into forward contracts. Consider the party paying floating. The first forward contract involves paying \$2.3 million and receiving \$3.5 million in four months. It has a value of  $1.2e^{-0.05 \times 4/12} = \$1.180$  million. To value the second forward contract, we note that the forward interest rate is 5% per annum with continuous compounding, or 5.063% per annum with semiannual compounding. The value of the forward contract is

$$100 \times (0.07 \times 0.5 - 0.05063 \times 0.5)e^{-0.05 \times 10/12} = \$0.929 \text{ million}$$

The total value of the forward contracts is therefore  $\$1.180 + \$0.929 = \$2.109$  million.

#### Problem 7.4.

*Explain what a swap rate is. What is the relationship between swap rates and par yields?*

A swap rate for a particular maturity is the average of the bid and offer fixed rates that a market maker is prepared to exchange for LIBOR in a standard plain vanilla swap with that maturity. The swap rate for a particular maturity is the LIBOR/swap par yield for that maturity.

#### Problem 7.5.

*A currency swap has a remaining life of 15 months. It involves exchanging interest at 10% on £20 million for interest at 6% on \$30 million once a year. The term structure of interest rates in both the United Kingdom and the United States is currently flat, and if the swap were negotiated today the interest rates exchanged would be 4% in dollars and 7% in sterling. All interest rates are quoted with annual compounding. The current exchange rate (dollars per pound sterling) is 1.8500. What is the value of the swap to the party paying sterling? What is the value of the swap to the party paying dollars?*

The swap involves exchanging the sterling interest of  $20 \times 0.10 = 2.0$  million for the dollar interest of  $30 \times 0.06 = \$1.8$  million. The principal amounts are also exchanged at the end of the life of the swap. The value of the sterling bond underlying the swap is

$$\frac{2}{(1.07)^{1/4}} + \frac{22}{(1.07)^{5/4}} = 22.182 \text{ million pounds}$$

The value of the dollar bond underlying the swap is

$$\frac{1.8}{(1.04)^{1/4}} + \frac{31.8}{(1.04)^{5/4}} = \$32.061 \text{ million}$$

The value of the swap to the party paying sterling is therefore

$$32.061 - (22.182 \times 1.85) = -\$8.976 \text{ million}$$

The value of the swap to the party paying dollars is +\$8.976 million. The results can also be obtained by viewing the swap as a portfolio of forward contracts. The continuously compounded interest rates in sterling and dollars are 6.766% per annum and 3.922% per annum. The 3-month and 15-month forward exchange rates are  $1.85e^{(0.03922-0.06766) \times 0.25} = 1.8369$  and  $1.85e^{(0.03922-0.06766) \times 1.25} = 1.7854$ . The values of the two forward contracts corresponding to the exchange of interest for the party paying sterling are therefore

$$(1.8 - 2 \times 1.8369)e^{-0.03922 \times 0.25} = -\$1.855 \text{ million}$$

$$(1.8 - 2 \times 1.7854)e^{-0.03922 \times 1.25} = -\$1.686 \text{ million}$$

The value of the forward contract corresponding to the exchange of principals is

$$(30 - 20 \times 1.7854)e^{-0.03922 \times 1.25} = -\$5.435 \text{ million}$$

The total value of the swap is  $-\$1.855 - \$1.686 - \$5.435 = -\$8.976$  million.

#### **Problem 7.6.**

*Explain the difference between the credit risk and the market risk in a financial contract.*

Credit risk arises from the possibility of a default by the counterparty. Market risk arises from movements in market variables such as interest rates and exchange rates. A complication is that the credit risk in a swap is contingent on the values of market variables. A company's position in a swap has credit risk only when the value of the swap to the company is positive.

#### **Problem 7.7.**

*A corporate treasurer tells you that he has just negotiated a five-year loan at a competitive fixed rate of interest of 5.2%. The treasurer explains that he achieved the 5.2% rate by borrowing at six-month LIBOR plus 150 basis points and swapping LIBOR for 3.7%. He goes on to say that this was possible because his company has a comparative advantage in the floating-rate market. What has the treasurer overlooked?*

The rate is not truly fixed because, if the company's credit rating declines, it will not be able to roll over its floating rate borrowings at LIBOR plus 150 basis points. The effective fixed borrowing rate then increases. Suppose for example that the treasurer's spread over LIBOR increases from 150 basis points to 200 basis points. The borrowing rate increases from 5.2% to 5.7%.

#### **Problem 7.8.**

*Explain why a bank is subject to credit risk when it enters into two offsetting swap contracts.*

At the start of the swap, both contracts have a value of approximately zero. As time passes, it is likely that the swap values will change, so that one swap has a positive value to the bank and the other has a negative value to the bank. If the counterparty on the other side of the positive-value swap defaults, the bank still has to honor its contract with the other counterparty. It is liable to lose an amount equal to the positive value of the swap.

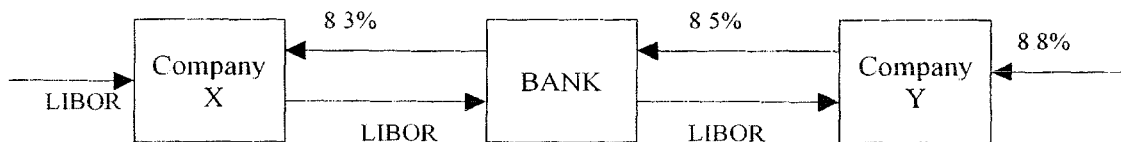
**Problem 7.9.**

Companies X and Y have been offered the following rates per annum on a \$5 million 10-year investment:

	Fixed Rate	Floating Rate
Company X	8.0%	LIBOR
Company Y	8.8%	LIBOR

Company X requires a fixed-rate investment; company Y requires a floating-rate investment. Design a swap that will net a bank, acting as intermediary, 0.2% per annum and will appear equally attractive to X and Y.

The spread between the interest rates offered to X and Y is 0.8% per annum on fixed rate investments and 0.0% per annum on floating rate investments. This means that the total apparent benefit to all parties from the swap is 0.8% per annum. Of this 0.2% per annum will go to the bank. This leaves 0.3% per annum for each of X and Y. In other words, company X should be able to get a fixed-rate return of 8.3% per annum while company Y should be able to get a floating-rate return LIBOR + 0.3% per annum. The required swap is shown in Figure S7.3. The bank earns 0.2%, company X earns 8.3%, and company Y earns LIBOR + 0.3%.



**Figure S7.3** Swap for Problem 7.9

**Problem 7.10.**

A financial institution has entered into an interest rate swap with company X. Under the terms of the swap, it receives 10% per annum and pays six-month LIBOR on a principal of \$10 million for five years. Payments are made every six months. Suppose that company X defaults on the sixth payment date (end of year 3) when the interest rate (with semiannual compounding) is 8% per annum for all maturities. What is the loss to the financial institution? Assume that six-month LIBOR was 9% per annum halfway through year 3.

At the end of year 3 the financial institution was due to receive \$500,000 ( $= 0.5 \times 10\%$  of \$10 million) and pay \$450,000 ( $= 0.5 \times 9\%$  of \$10 million). The immediate loss is therefore \$50,000. To value the remaining swap we assume that forward rates are realized. All forward rates are 8% per annum. The remaining cash flows are therefore valued on the assumption that the floating payment is  $0.5 \times 0.08 \times 10,000,000 = \$400,000$  and the net payment that would be received is  $500,000 - 400,000 = \$100,000$ . The total cost of default is therefore the cost of foregoing the following cash flows:

year 3:	\$50,000
year 3½:	\$100,000
year 4:	\$100,000
year 4½:	\$100,000
year 5:	\$100,000

Discounting these cash flows to year 3 at 4% per six months we obtain the cost of the default as \$413,000.

**Problem 7.11.**

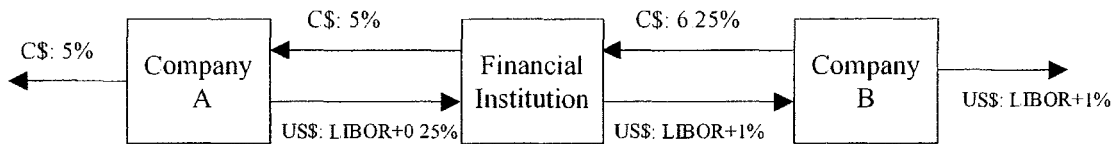
Companies A and B face the following interest rates (adjusted for the differential impact of taxes):

	A	B
U.S. dollars (floating rate)	LIBOR + 0.5%	LIBOR + 1.0%
Canadian dollars (fixed rate)	5.0%	6.5%

Assume that A wants to borrow U.S. dollars at a floating rate of interest and B wants to borrow Canadian dollars at a fixed rate of interest. A financial institution is planning to arrange a swap and requires a 50-basis-point spread. If the swap is equally attractive to A and B, what rates of interest will A and B end up paying?

Company A has a comparative advantage in the Canadian dollar fixed-rate market. Company B has a comparative advantage in the U.S. dollar floating-rate market. (This may be because of their tax positions.) However, company A wants to borrow in the U.S. dollar floating-rate market and company B wants to borrow in the Canadian dollar fixed-rate market. This gives rise to the swap opportunity.

The differential between the U.S. dollar floating rates is 0.5% per annum, and the differential between the Canadian dollar fixed rates is 1.5% per annum. The difference between the differentials is 1% per annum. The total potential gain to all parties from the swap is therefore 1% per annum, or 100 basis points. If the financial intermediary requires 50 basis points, each of A and B can be made 25 basis points better off. Thus a swap can be designed so that it provides A with U.S. dollars at LIBOR + 0.25% per annum, and B with Canadian dollars at 6.25% per annum. The swap is shown in Figure S7.4.



**Figure S7.4** Swap for Problem 7.11

Principal payments flow in the opposite direction to the arrows at the start of the life of the swap and in the same direction as the arrows at the end of the life of the swap. The financial institution would be exposed to some foreign exchange risk which could be hedged using forward contracts.

**Problem 7.12.**

A financial institution has entered into a 10-year currency swap with company Y. Under the terms of the swap, the financial institution receives interest at 3% per annum in Swiss francs and pays interest at 8% per annum in U.S. dollars. Interest payments are exchanged once a year. The principal amounts are 7 million dollars and 10 million francs. Suppose that company Y declares bankruptcy at the end of year 6, when the exchange rate is \$0.80 per franc. What is the cost to the financial institution? Assume that, at the end of year 6, the interest rate is 3% per annum in Swiss francs and 8% per annum in U.S. dollars for all maturities. All interest rates are quoted with annual compounding.

When interest rates are compounded annually

$$F_0 = S_0 \left( \frac{1 + r}{1 + r_f} \right)^T$$

where  $F_0$  is the  $T$ -year forward rate,  $S_0$  is the spot rate,  $r$  is the domestic risk-free rate, and  $r_f$  is the foreign risk-free rate. As  $r = 0.08$  and  $r_f = 0.03$ , the spot and forward exchange rates at the end of year 6 are

spot:	0.8000
1 year forward:	0.8388
2 year forward:	0.8796
3 year forward:	0.9223
4 year forward:	0.9670

The value of the swap at the time of the default can be calculated on the assumption that forward rates are realized. The cash flows lost as a result of the default are therefore as follows:



Year	Dollar Paid	Swiss Franc Received	Forward Rate	Dollar Equivalent of Swiss Franc Received	Cash Flow Lost
6	560,000	300,000	0.8000	240,000	(320,000)
7	560,000	300,000	0.8388	251,600	(308,400)
8	560,000	300,000	0.8796	263,900	(296,100)
9	560,000	300,000	0.9223	276,700	(283,300)
10	7,560,000	10,300,000	0.9670	9,960,100	2,400,100

Discounting the numbers in the final column to the end of year 6 at 8% per annum, the cost of the default is \$679,800.

Note that, if this were the only contract entered into by company Y, it would make no sense for the company to default at the end of year six as the exchange of payments at that time has a positive value to company Y. In practice company Y is likely to be defaulting and declaring bankruptcy for reasons unrelated to this particular contract and payments on the contract are likely to stop when bankruptcy is declared.

**Problem 7.13.**

*After it hedges its foreign exchange risk using forward contracts, is the financial institution's average spread in Figure 7.10 likely to be greater than or less than 20 basis points? Explain your answer.*

The financial institution will have to buy 1.1% of the AUD principal in the forward market for each year of the life of the swap. Since AUD interest rates are higher than dollar interest rates, AUD is at a discount in forward markets. This means that the AUD purchased for year 2 is less expensive than that purchased for year 1; the AUD purchased for year 3 is less expensive than that purchased for year 2; and so on. This works in favor of the financial institution and means that its spread increases with time. The spread is always above 20 basis points.

**Problem 7.14.**

*"Companies with high credit risks are the ones that cannot access fixed-rate markets directly. They are the companies that are most likely to be paying fixed and receiving floating in an interest rate swap." Assume that this statement is true. Do you think it increases or decreases the risk of a financial institution's swap portfolio? Assume that companies are most likely to default when interest rates are high.*

Consider a plain-vanilla interest rate swap involving two companies X and Y. We suppose that X is paying fixed and receiving floating while Y is paying floating and receiving fixed.

The quote suggests that company X will usually be less creditworthy than company Y. (Company X might be a BBB-rated company that has difficulty in accessing fixed-rate markets directly; company Y might be a AAA-rated company that has no difficulty accessing fixed or floating rate markets.) Presumably company X wants fixed-rate funds and company Y wants floating-rate funds.

The financial institution will realize a loss if company Y defaults when rates are high or if company X defaults when rates are low. These events are relatively unlikely since (a)

Y is unlikely to default in any circumstances and (b) defaults are less likely to happen when rates are low. For the purposes of illustration, suppose that the probabilities of various events are as follows:

Default by Y:	0.001
Default by X:	0.010
Rates high when default occurs:	0.7
Rates low when default occurs:	0.3

The probability of a loss is

$$0.001 \times 0.7 + 0.010 \times 0.3 = 0.0037$$

If the roles of X and Y in the swap had been reversed the probability of a loss would be

$$0.001 \times 0.3 + 0.010 \times 0.7 = 0.0073$$

Assuming companies are more likely to default when interest rates are high, the above argument shows that the observation in quotes has the effect of decreasing the risk of a financial institution's swap portfolio. It is worth noting that the assumption that defaults are more likely when interest rates are high is open to question. The assumption is motivated by the thought that high interest rates often lead to financial difficulties for corporations. However, there is often a time lag between interest rates being high and the resultant default. When the default actually happens interest rates may be relatively low.

**Problem 7.15.**

*Why is the expected loss from a default on a swap less than the expected loss from the default on a loan with the same principal?*

In an interest-rate swap a financial institution's exposure depends on the difference between a fixed-rate of interest and a floating-rate of interest. It has no exposure to the notional principal. In a loan the whole principal can be lost.

**Problem 7.16.**

*A bank finds that its assets are not matched with its liabilities. It is taking floating-rate deposits and making fixed-rate loans. How can swaps be used to offset the risk?*

The bank is paying a floating-rate on the deposits and receiving a fixed-rate on the loans. It can offset its risk by entering into interest rate swaps (with other financial institutions or corporations) in which it contracts to pay fixed and receive floating.

**Problem 7.17.**

*Explain how you would value a swap that is the exchange of a floating rate in one currency for a fixed rate in another currency.*

The floating payments can be valued in currency A by (i) assuming that the forward rates are realized, and (ii) discounting the resulting cash flows at appropriate currency A discount rates. Suppose that the value is  $V_A$ . The fixed payments can be valued in

currency B by discounting them at the appropriate currency B discount rates. Suppose that the value is  $V_B$ . If  $Q$  is the current exchange rate (number of units of currency A per unit of currency B), the value of the swap in currency A is  $V_A - QV_B$ . Alternatively, it is  $V_A/Q - V_B$  in currency B.

**Problem 7.18.**

The LIBOR zero curve is flat at 5% (continuously compounded) out to 1.5 years. Swap rates for 2- and 3-year semiannual pay swaps are 5.4% and 5.6%, respectively. Estimate the LIBOR zero rates for maturities of 2.0, 2.5, and 3.0 years. (Assume that the 2.5-year swap rate is the average of the 2- and 3-year swap rates.)

The two-year swap rate is 5.4%. This means that a two-year LIBOR bond paying a semiannual coupon at the rate of 5.4% per annum sells for par. If  $R_2$  is the two-year LIBOR zero rate

$$2.7e^{-0.05 \times 0.5} + 2.7e^{-0.05 \times 1.0} + 2.7e^{-0.05 \times 1.5} + 102.7e^{-R_2 \times 2.0} = 100$$

Solving this gives  $R_2 = 0.05342$ . The 2.5-year swap rate is assumed to be 5.5%. This means that a 2.5-year LIBOR bond paying a semiannual coupon at the rate of 5.5% per annum sells for par. If  $R_{2.5}$  is the 2.5-year LIBOR zero rate

$$2.75e^{-0.05 \times 0.5} + 2.75e^{-0.05 \times 1.0} + 2.75e^{-0.05 \times 1.5} + 2.75e^{-0.05342 \times 2.0} + 102.75e^{-R_{2.5} \times 2.5} = 100$$

Solving this gives  $R_{2.5} = 0.05442$ . The 3-year swap rate is 5.6%. This means that a 3-year LIBOR bond paying a semiannual coupon at the rate of 5.6% per annum sells for par. If  $R_3$  is the three-year LIBOR zero rate

$$2.8e^{-0.05 \times 0.5} + 2.8e^{-0.05 \times 1.0} + 2.8e^{-0.05 \times 1.5} + 2.8e^{-0.05342 \times 2.0} + 2.8e^{-0.05442 \times 2.5} + 102.8e^{-R_3 \times 3.0} = 100$$

Solving this gives  $R_3 = 0.05544$ . The zero rates for maturities 2.0, 2.5, and 3.0 years are therefore 5.342%, 5.442%, and 5.544%, respectively.

## ASSIGNMENT QUESTIONS

**Problem 7.19.**

The one-year LIBOR rate is 10%. A bank trades swaps where a fixed rate of interest is exchanged for 12-month LIBOR with payments being exchanged annually. Two- and three-year swap rates (expressed with annual compounding) are 11% and 12% per annum. Estimate the two- and three-year LIBOR zero rates.

The two-year swap rate implies that a two-year LIBOR bond with a coupon of 11% sells for par. If  $R_2$  is the two-year zero rate

$$11e^{-0.10 \times 1.0} + 111e^{-R_2 \times 2.0} = 100$$

so that  $R_2 = 0.1046$  The three-year swap rate implies that a three-year LIBOR bond with a coupon of 12% sells for par. If  $R_3$  is the three-year zero rate

$$12e^{-0.10 \times 1.0} + 12e^{-0.1046 \times 2.0} + 112e^{-R_3 \times 3.0} = 100$$

so that  $R_3 = 0.1146$  The two- and three-year rates are therefore 10.46% and 11.46% with continuous compounding.

**Problem 7.20.**

Company A, a British manufacturer, wishes to borrow U.S. dollars at a fixed rate of interest. Company B, a U.S. multinational, wishes to borrow sterling at a fixed rate of interest. They have been quoted the following rates per annum (adjusted for differential tax effects):

	Sterling	U.S. Dollars
Company A	11.0%	7.0%
Company B	10.6%	6.2%

Design a swap that will net a bank, acting as intermediary, 10 basis points per annum and that will produce a gain of 15 basis points per annum for each of the two companies.

The spread between the interest rates offered to A and B is 0.4% (or 40 basis points) on sterling loans and 0.8% (or 80 basis points) on U.S. dollar loans. The total benefit to all parties from the swap is therefore

$$80 - 40 = 40 \text{ basis points}$$

It is therefore possible to design a swap which will earn 10 basis points for the bank while making each of A and B 15 basis points better off than they would be by going directly to financial markets. One possible swap is shown in Figure M7.1. Company A borrows at an effective rate of 6.85% per annum in U.S. dollars.

Company B borrows at an effective rate of 10.45% per annum in sterling. The bank earns a 10-basis-point spread. The way in which currency swaps such as this operate is as follows. Principal amounts in dollars and sterling that are roughly equivalent are chosen. These principal amounts flow in the opposite direction to the arrows at the time the swap is initiated. Interest payments then flow in the same direction as the arrows during the life of the swap and the principal amounts flow in the same direction as the arrows at the end of the life of the swap.

Note that the bank is exposed to some exchange rate risk in the swap. It earns 65 basis points in U.S. dollars and pays 55 basis points in sterling. This exchange rate risk could be hedged using forward contracts.

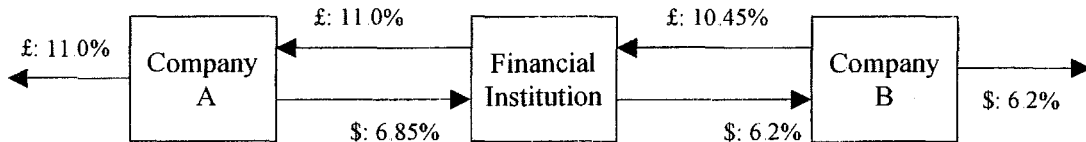


Figure M7.1 One Possible Swap for Problem 7.20

**Problem 7.21.**

Under the terms of an interest rate swap, a financial institution has agreed to pay 10% per annum and to receive three-month LIBOR in return on a notional principal of \$100 million with payments being exchanged every three months. The swap has a remaining life of 14 months. The average of the bid and offer fixed rates currently being swapped for three-month LIBOR is 12% per annum for all maturities. The three-month LIBOR rate one month ago was 11.8% per annum. All rates are compounded quarterly. What is the value of the swap?

The swap can be regarded as a long position in a floating-rate bond combined with a short position in a fixed-rate bond. The correct discount rate is 12% per annum with quarterly compounding or 11.82% per annum with continuous compounding.

Immediately after the next payment the floating-rate bond will be worth \$100 million. The next floating payment (\$ million) is

$$0.118 \times 100 \times 0.25 = 2.95$$

The value of the floating-rate bond is therefore

$$102.95e^{-0.1182 \times 2/12} = 100.941$$

The value of the fixed-rate bond is

$$2.5e^{-0.1182 \times 2/12} + 2.5e^{-0.1182 \times 5/12} + 2.5e^{-0.1182 \times 8/12} \\ + 2.5e^{-0.1182 \times 11/12} + 102.5e^{-0.1182 \times 14/12} = 98.678$$

The value of the swap is therefore

$$100.941 - 98.678 = \$2.263 \text{ million}$$

As an alternative approach we can value the swap as a series of forward rate agreements. The calculated value is

$$(2.95 - 2.5)e^{-0.1182 \times 2/12} + (3.0 - 2.5)e^{-0.1182 \times 5/12} \\ + (3.0 - 2.5)e^{-0.1182 \times 8/12} + (3.0 - 2.5)e^{-0.1182 \times 11/12} \\ + (3.0 - 2.5)e^{-0.1182 \times 14/12} = \$2.263 \text{ million}$$

which is in agreement with the answer obtained using the first approach.

**Problem 7.22.**

Suppose that the term structure of interest rates is flat in the United States and Australia. The USD interest rate is 7% per annum and the AUD rate is 9% per annum. The current value of the AUD is 0.62 USD. Under the terms of a swap agreement, a financial institution pays 8% per annum in AUD and receives 4% per annum in USD. The principals in the two currencies are \$12 million USD and 20 million AUD. Payments are exchanged every year, with one exchange having just taken place. The swap will last two more years. What is the value of the swap to the financial institution? Assume all interest rates are continuously compounded.

The financial institution is long a dollar bond and short a USD bond. The value of the dollar bond (in millions of dollars) is

$$0.48e^{-0.07 \times 1} + 12.48e^{-0.07 \times 2} = 11.297$$

The value of the AUD bond (in millions of AUD) is

$$1.6e^{-0.09 \times 1} + 21.6e^{-0.09 \times 2} = 19.504$$

The value of the swap (in millions of dollars) is therefore

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$$11.297 - 19.504 \times 0.62 = -0.795$$

or -\$795,000.

As an alternative we can value the swap as a series of forward foreign exchange contracts. The one-year forward exchange rate is  $0.62e^{-0.02} = 0.6077$ . The two-year forward exchange rate is  $0.62e^{-0.02 \times 2} = 0.5957$ . The value of the swap in millions of dollars is therefore

$$(0.48 - 1.6 \times 0.6077)e^{-0.07 \times 1} + (12.48 - 21.6 \times 0.5957)e^{-0.07 \times 2} = -0.795$$

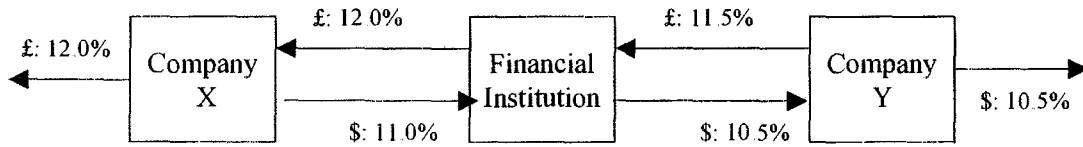
which is in agreement with the first calculation.

**Problem 7.23.**

Company X is based in the United Kingdom and would like to borrow \$50 million at a fixed rate of interest for five years in U.S. funds. Because the company is not well known in the United States, this has proved to be impossible. However, the company has been quoted 12% per annum on fixed-rate five-year sterling funds. Company Y is based in the United States and would like to borrow the equivalent of \$50 million in sterling funds for five years at a fixed rate of interest. It has been unable to get a quote but has been offered U.S. dollar funds at 10.5% per annum. Five-year government bonds currently yield 9.5% per annum in the United States and 10.5% in the United Kingdom. Suggest an appropriate currency swap that will net the financial intermediary 0.5% per annum.

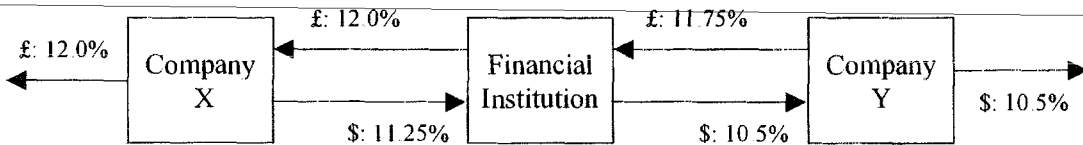
There is a 1% differential between the yield on sterling and dollar 5-year bonds. The financial intermediary could use this differential when designing a swap. For example, it

could (a) allow company X to borrow dollars at 1% per annum less than the rate offered on sterling funds, that is, at 11% per annum and (b) allow company Y to borrow sterling at 1% per annum more than the rate offered on dollar funds, that is, at  $11\frac{1}{2}\%$  per annum. However, as shown in Figure M7.2, the financial intermediary would not then earn a positive spread.



**Figure M7.2** First attempt at designing swap for Problem 7.23

To make 0.5% per annum, the financial intermediary could add 0.25% per annum, to the rates paid by each of X and Y. This means that X pays 11.25% per annum, for dollars and Y pays 11.75% per annum, for sterling and leads to the swap shown in Figure M7.3. The financial intermediary would be exposed to some foreign exchange risk in this swap. This could be hedged using forward contracts.



**Figure M7.3** Final Swap for Problem 7.23